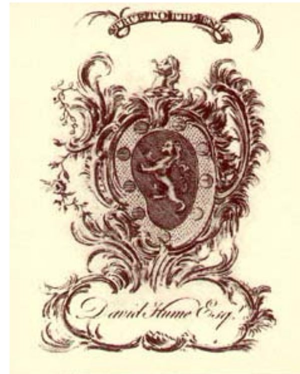

Hume's Geometric 'Objects'

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HUME'S GEOMETRIC "OBJECTS"

Arithmetic and algebra allow of precision and certainty. The science of geometry is not likewise a perfect and infallible science. At any rate, this is Hume's teaching in the Treatise.

> *When two numbers are so combin'd, as that the one has always an unite answering to every unite of the other, we pronounce them equal; and 'tis for want of such a standard of equality in extension, that geometry can scarce be esteem'd a perfect and infallible science.*[1]

The ideas of, say, equality and right line shade into inequality and curve. It must not be supposed, however, that the vagueness of these ideas renders all judgments uncertain and fallible. *'Tis evident, that the eye, or rather the mind is often able at one view to determine the proportions of bodies....Such judgments are not only common, but in many cases certain and infallible.* (T47) Nevertheless, geometry is a fallible science because its essential ideas are vague. Vagueness, due to lack of boundaries between ideas, is in the ideas themselves. For this reason, *'tis impossible to produce any definition of them, which will fix the precise boundaries betwixt them.* (T49) (Yet, some forty pages earlier Hume argued at length *that the mind cannot form any notion of quantity or quality without forming a precise notion of the degrees of each* [T18])

As far as the Treatise goes, geometry is the science of measurement and as such it must allow for error. Even if precise geometric standards could be had, they would not make the propositions of geometry certain. Thus, we might precisely define a right line by a rule of order of indivisible points. But geometry, construed as the science of measurement, would not be perfected.

> *Nor, if it were* (the standard), *is there any such firmness in our senses or imagination, as to determine when such an order is violated or preserv'd. The original standard of a right line is in reality nothing but a certain general appearance.* (T52)

In the Enquiries, Hume continues to hold that notions essential to geometry, for example, equality, right line, and so on, are sensibles. But they are no longer generic appearances. They are determinate.

> *The great advantage of the mathematical sciences above the moral consists in this, that the ideas of the former, being sensible, are always clear and determinate...even when no definition is employed, the object itself may be presented to the senses, and by that means be steadily and clearly apprehended.*[2]

In addition, geometry is no longer merely the science of measurement. The truths of Euclid, Hume holds, would remain truths even if there were no figures in nature. *Though there never were a circle or triangle in nature, the truths demonstrated by Euclid would for ever retain their certainty and evidence.* (EHU25)

Professor Zabeeh, in his commentary on Hume, holds that Hume did not mean to imply that Euclid could dispense with the observation of figures.[3] However, whenever possible Hume should be taken at his word: observation of figures is not a truth condition for propositions of geometry. A distinction must be drawn between discovery conditions and what Hume calls the "truth or evidence" of a geometric proposition. We may suppose that observation of figures is a discovery condition. But it is hardly a truth condition. The certainty, as opposed to the discovery, of geometric propositions does not depend on the existence in nature of any figure.

I am mainly concerned, however, with Professor Zabeeh's account of how, according to Hume, geometric propositions retain their *certainty and evidence* in the face of recalcitrant experience. Noting that Hume does not "fully explain," he gives the real reason for Hume's belief in the apodeictic certainty of mathematical truths.

> He appears to assume we never allow such truths to be controverted by empirical evidence. That is to say, if perchance we find, by measurement, that the sum of the angles of a Euclidean triangle

> does not equal 180 degrees, either we say that we measured wrongly or we say that the triangle we have been measuring is not Euclidean.[4]

This formulation of Hume's reason avoids the distinction between pure and applied geometry, a distinction which, according to Professor Zabeeh, Hume refuses to draw.[5] In what follows, I try to show that Hume's shift in view in the Enquiries to clear, determinate geometric ideas, together with the distinction between idea and existence in nature, suffices for the distinction. The truths of Euclid are truths about abstract geometric ideas. They remain truths whether or not anything in nature corresponds to them, i.e. geometry is not merely the science of measurement. If so, geometric truths face no recalcitrant experience.

The "objects" of mathematicians, according to Hume, are ordinarily taken to be of a refined and spiritual nature.

> *'Tis usual with mathematicians, to pretend, that those ideas, which are their objects, are of so refin'd and spiritual a nature, that they fall not under the conception of the fancy, but must be comprehended by a pure and intellectual view, of which the superior faculties of the soul are alone capable.* (T72)

Geometers appear driven to refined and spiritual "objects" because, in their view, no sensible figure can be perfect. Sensible circles always fall short of the geometer's definition of 'circle'. Regular figures composed entirely of straight lines, if sensible, are likewise imperfect, since no sensible line can be perfectly straight. Apparently equal figures turn out to differ on closer examination. The notions of absolute equality, straight line, perfect circle, and so on, are sense transcendent archetypes.

Hume disagrees. The ideas of geometric figures, of straight line, of equality, and so on, are sensible. He begins his account by inquiring whether or not indivisible points -- minima sensibilia -- provide a basis for a definitional standard for geometric equality? Two bodies are equal, we might suppose, if, and only if, addition or removal of a point renders them unequal. Hume rejects this proposal on the ground that gain or loss of a minimum sensible makes no detectable difference either before or after the most refined measurement of two apparently equal bodies. (T45) *The only useful notion of equality, or inequality,* he holds, *is deriv'd from the whole united appearance and the comparison of particular objects.* (T637)

How, then, does Hume account for the (mistaken) view that such notions as equality are transcendent? Correction by addition or subtraction of a quantity is an "action of the mind" which, when merely imagined, carries one beyond what sense, even sense aided by art and instruments, can discern. This action of the mind, being imaginary, abandons the "just and useful" idea of sensible equality. Imagined corrections terminate in a fictive equality. The latter is as useless as it is incomprehensible.[6]

Hume's account is clear enough. Corrections up to a sensible limit yield perfect geometric ideas. However, he does nothing to meet a classic objection to his account. In what follows, I try to show that the objection fails. The argument is developed in terms of the notion of perfect circle. Thereafter, the notions of equality and straight line are considered.

Let us grant that a circle appears to satisfy the geometer's definition of 'circle'. It is the sensible limit to the process of correction. However, Hume's claim that the appearance of such a circle is the original of the corresponding idea does not follow. The very concept of correction pre-supposes adoption and use of a standard, enshrined in the geometer's definition of 'circle', in terms of which one determines that one circle is rounder than another circle. It is rounder if and only if the former is a correction of the latter, relative to a standard to which both are brought for comparison. The standard is a perfect circle, <u>i.e.</u>, the circle specified by the geometer's definition of 'circle'. Because one makes corrections, so the objection goes, one must have this idea of a perfect circle. If the latter is sense transcendent, the mathematicians are right and Hume is wrong. Even if it is not sense transcendent, per impossible, Hume's account is fatally flawed. It is viciously circular. The process of correction presupposes use of the very idea of perfect circle which it is alleged to generate.

Suppose, however, that two different circles are brought to a third circle, and, on comparison, one is rounder than the other, relative to a third circle. It does not follow that the former is a "correction" of the latter. All that follows, strictly speaking, is that, of two circles standing to a third circle, the first is more like and the second is less like the third. 'Rounder than' specifies the respect in which objects are to be compared. They are to be compared in respect to their circularity. It specifies, also, the outcome of their comparison in that respect, i.e., the first is more like and the second is less like the third. The notion of the first being a "correction" of the second is logically adventitious to comparison for degree of circularity. However, let us grant, for the sake of argument, that circles do "correct" one another relative to a standard circle. It not only does not follow, it is just false, that the standard must be a perfect circle, i.e., a circle satisfying the geometer's definition of 'circle'. Thus, I may have no idea of a perfect circle. Yet, given three different circles, none perfect, I compare them and discover that the first is more like and the second is less like the third. Given suitable circles, I may serialize them, as follows: the role of standard passes from item to item as the subject of the generating relation ('more like') changes. Thus, in the case of $circle_1$ being more like $circle_2$ than it is like $circle_3$, $circle_1$ functions as standard. $Circle_2$ and $circle_3$ are brought to the standard for comparison. On comparison, $circle_1$ and $circle_2$ are more like than are $circle_1$ and $circle_3$. Now assume that $circle_3$ is more like $circle_2$ than it is like $circle_1$. In this case, $circle_3$ is the standard. On comparison, $circle_3$ and $circle_2$ are more like than are $circle_3$ and $circle_1$. The outcome is a series, <u>i.e.</u>, $circle_2$ lies between $circle_1$ and $circle_3$. The series so generated depends on shifting standards, in this case, from $circle_1$ to $circle_3$. By expanding to new items and repeating comparisons, the series is enlarged. None of the items in the enlarged series, however, is necessarily the circle specified by the geometer's definition of 'circle' and, more to the point, one could know the items in the series and know how the series is generated without having or using the idea of a perfect circle.

As we have just seen, the notion of one item in a series being the "correction" of another item in that series is logically adventitious to any series whose generating relation is 'more like'. However, the concept of correction could be built into the notion of such a series. Assuming it is built in, the objection to Hume's account of our ideas of perfect standards nonetheless fails. It does not follow from the fact that one can make corrections, that one must correct for a perfect standard. One could "correct" up to a sensible limit without having the idea of a perfect standard. Upon reaching the limit, that item at the limit is assigned perfect status.

The foregoing argument holds for any geometric notion susceptible to degrees. We do say that one line is straighter than another line -- showing that lines may be compared for degree of straightness. However, an alternative argument works as well for straight line; indeed, for any geometric notion including equality. Consider three figures, F_1, F_2 and F_3, of which no apparent difference lies between F_1 and F_2, whereas the difference between F_2 and F_3 is apparent. Using these figures, one might teach or learn the logical contrast between equality and inequality. Through this contrast between equality and inequality one acquires their corresponding ideas. If it should turn out that F_1 and F_2 actually differ, then the belief that they are equal is mistaken. Nevertheless, the idea of figure equality is acquired. All Hume would insist on is this, that we are not always mistaken, that our judgments of equality are *in many cases certain and infallible.*

If such ideas as perfect circle, equality and straight line are plausibly regarded as sensible, no obstacle remains to accepting all other ideas essential to geometry as sensible. As Hume states in the Enquiries, as noted above, where a definition is lacking, the figure itself may be presented.

Even in the Treatise, Hume argues for ultimate, i.e., perfect sensible geometric standards.

> *As the ultimate standard of these figures is deriv'd from nothing but the senses and imagination, 'tis absurd to talk of any perfection beyond what these faculties can judge of; since the true perfection of any thing consists in its conformity to its standard.* (T51)

However, in the Treatise Hume is still spellbound by the picture of geometry as a quasi-empirical science. Holding that where there are no differences or distinctions, determinate ideas cannot be of any use in drawing them, he continues to insist on generic ideas and the weakness and fallibility of geometry. The change in his view in the Enquiries consists in moving to clear and determinate ideas and reconstruing geometric propositions as true of the world just to the extent that the world matches these clear and determinate ideas. Whether or not there is a match, the propositions remain certain.

Hume's geometric "objects" are mainly figures which are themselves abstract "aspects" of concrete particular ideas, as set forth in the doctrine of distinctions of reason. (T24-25) Aspects, on this doctrine, are resemblances among different particular ideas and these resemblances provide the basis for the generality of general words. (T20ff) Thus, ideas are for Hume figures. They may be, for example, widely different triangles. Nevertheless, they are united by a resemblance which governs ordinary use of 'triangle'. Perhaps this resemblance is enshrined in the geometer's definition of 'triangle', and, if so, it would be a mistake to suppose in this case that what the ordinary man thinks of in using 'triangle' differs from the geometer's "object" as he defines 'triangle'. However, Hume distinguishes between "objects" and reflection and reasoning about them, and I have tried to preserve this distinction in discussing Hume's geometric "objects." For I have had nothing to say about his theory or theories of reflection and reasoning. Rather, my problem has been to show how such reflection and reasoning retains its apodeictic certainty even if nature were devoid of figures and, yet, how such reasoning applies to figures in nature. The solution lies in Hume's theory of sensible geometric "objects."

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1. L. A. Selby-Bigge, ed., Hume's Treatise (Oxford: Clarendon Press, 1958), p.71.

2. L. A. Selby-Bigge, ed., Hume's Enquiries (Oxford: Clarendon Press, 1951), p.60.

3. Farhang Zabeeh, Hume: Precursor of Modern Empiricism (The Hague: Martinus Nijhoff, 1960), p.142.

4. Ibid., p.143.

5. Ibid.,

6. *The notion of any correction beyond what we have instruments and art to make, is a mere fiction of the mind, and useless as well as incomprehensible.* (T48)