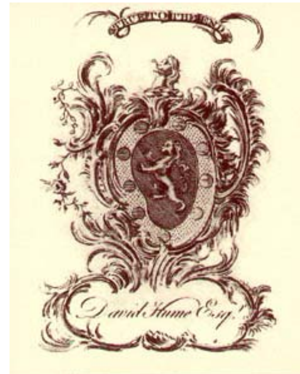

Hume on Space and Geometry

Rosemary Newman

Hume Studies Volume VII, Number 1 (April, 1981), 1-31.

HUME ON SPACE AND GEOMETRY

Hume's discussion of our ideas of space, time and mathematics in Book One of the Treatise is referred to by one recent commentator as 'the least admired part' of this work,[1] while another finds it to be 'one of the least satisfactory Parts'.[2] Hume himself, it would appear, was not far from endorsing such opinions. The omission of any detailed comment on these subjects from the first Enquiry can be taken as indicating his dissatisfaction with the earlier exposition as well as a felt incompetence to offer a suitable revision. For there was certainly no loss of interest on Hume's part in the philosophical problems of mathematics. In a letter to William Strahan in 1772 he speaks of an essay prepared for publication around 1755 entitled: *On the Metaphysical Principles of Geometry*, which he withdrew upon the advice of his friend the mathematician Lord Stanhope, who *convinced me that either there was some defect in the argument or in its perspicuity; I forget which...*[3]

Defects of both kinds are present in the Treatise discussion of geometry and the infinite divisibility of space, which taken together with certain remarks in the Abstract of the Treatise and in the Enquiry[4], suggest that the problem defying satisfactory solution was the relation between these. Hume was of the opinion that the theory of infinite divisibility constituted an affront to human reason by virtue of the paradoxes it involved. Such paradoxes, he thought, must open the door to scepticism about the certainty of mathematical knowledge, since in the field of abstract reasoning reason is *thrown into a kind of amazement and suspence, which, without the suggestions of any sceptic, gives her a diffidence of herself, and of the ground on which she treads*. (E157) Hume's attempted solution involved the rejection of those abstract ideas employed in mathematics and most fundamentally, that of infinite divisibility, but he never succeeded in working out a theory of geometry which preserved its desired status as a body of certain knowledge and yet was at the same time consistent with his empiricism.

In this paper I set out to examine the relation between Hume's view of space and his theory of geometry. Firstly I take a detailed look at Hume's arguments against infinite divisibility in an attempt to discover why they take the form they do, and to understand what Hume has in mind when he speaks of that 'minimum' idea which is an 'adequate representation' of the most minute possible part of extension. The second section deals with Hume's account of the genesis of our idea of space. I propose, contrary to some commentators, that Hume does not treat the idea of space on an exact parallel with that of time. Although both are said to be abstract ideas they differ in the relation in which they first stand to impressions. My comments on time are confined to drawing attention to this difference since fuller discussion of passages dealing exclusively with time would require a separate paper. In the final section Hume's remarks about geometry are examined. Here I argue that Hume entertains two views of geometry in the Treatise, one of which is abandoned in the Enquiry while the other remains largely unaltered.

1

Hume opens his account of our ideas of space and time with an attack on the thesis of infinite divisibility which he regards as belonging to that body of doctrines which are *greedily embrac'd by philosophers, as shewing the superiority of their science, which cou'd discover opinions so remote from vulgar conception.* (T26) Two principles, both taken to be evident, form the basis of his argument. I shall refer to them hereafter as principles A and B.

Principle A: *the capacity of the mind is limited, and can never attain a full and adequate conception of infinity.*

Principle B: *whatever is capable of being divided in infinitum, must consist of an infinite number of parts, and ( ) 'tis impossible to set any bounds to the number of parts, without setting bounds at the same time to the division.* (T26, 27)

From these two principles Hume draws the conclusion that no idea we possess of any finite quality admits of infinite division. Repeated division will eventually result in a finite number of simple, indivisible ideas. This, he thinks, establishes beyond doubt that *the imagination reaches a minimum, and may raise up to itself an idea, of which it cannot conceive any subdivision, and which cannot be diminished without a total annihilation.* (T27)

Following on this comes the assertion that the minimum which lies at the threshold of the imagination's capacity for dividing ideas serves as an 'adequate idea' of the smallest possible part of any object. A similar claim is made about a minimum sensible impression, although with certain qualifications added. Hume sums up:

> *We may hence discover the error of the common opinion, that the capacity of the mind is limited on both sides, and that 'tis impossible for the imagination to form an adequate idea, of what goes beyond a certain degree of minuteness as well as of greatness. Nothing can be more minute, than some ideas, which we form in the fancy; and images, which appear to the senses; since there are ideas and images perfectly simple and indivisible.* (T28)

In the following section Hume applies the results of his reasoning to the concepts of space and time as proof that the most minute parts of extension and duration cannot be more minute than our most minute ideas of them. Then, taking up a minimum conceivable idea of extension he sets out to demonstrate, by repetition ad infinitum of this idea, that it is contradictory to suppose that a finite extension can contain an infinite number of parts because addition ad infinitum must result in an infinite extension. Armed with the additional principle that where 'adequate ideas' are concerned, *whatever appears impossible and contradictory upon the comparison of these ideas, must be really impossible and contradictory* (T29), Hume arrives at the conclusion that no finite extension is infinitely divisible. (T30) The nature of space (and time) must accord with our conception of it. That is: *tho' divisible into parts or inferior ideas,* (it) *is not infinitely divisible, nor consists of an infinite number of parts.* (T32)

Leaving to one side the question of the status of Hume's assertion that there must be conformity between the nature of reality and our 'adequate ideas', (which if it is not merely a trivial analytic truth must surely be regarded as a metaphysical postulate), the argument in Section Two clearly turns upon our possession of what Hume calls an 'adequate idea' of a minimal part of extension. Since this claim is only a particular application of a general conclusion reached in Section One, the strength of Hume's case (such as it is) must reside in the latter. It is this to which I now turn.

It is not surprising to find Hume launching his attack on the thesis of infinite divisibility with the claim that we have no *full and adequate* idea of infinity. The concepts of infinity and infinite number had exercised philosophers from earliest times, with Aristotle's distinction between an actual infinite and a potential infinite proving influential in its rejection of the latter notion. It is this contrast between the ideas of an actual infinite and of something as merely potentially infinite which Hume appears to have in mind in stating Principle A. There is a close resemblance here to Locke's assertion, in the Essay, that we have no positive idea of infinity but only the negative notion of '...a growing fugitive idea, still in a boundless progression, that can stop nowhere.[5] Locke regarded infinity as essentially a mathematical idea, which gains application to space and time through our conception of these as composed of parts, and although he thinks that 'endless addition or addibility (   ) of numbers....gives us the clearest and most distinct idea of infinity',[6] he denies that this can yield 'a clear and distinct idea of an actually infinite number'.[7]

In a similar vein, Hume, in putting forward Principle A, is not denying that we have an idea of infinity; rather, he is asserting that the idea of a potential infinity cannot suffice to support the implications of the thesis of infinite divisibility. The nature of the alleged deficiency is laid out in Principle B, which reveals Hume to be of the opinion that the concept of infinite divisibility has application solely to composites whose parts number an actual infinity. From this it follows that infinite divisibility understood in terms of a potentially infinite series of divisions and sub-divisions must presuppose the concept of an actual infinite number and cannot be coherently conceived independently of the coherence of that concept. And yet, *'tis universally allow'd* that we can have no clear conception of an actual infinity (of parts). Moreover and worse still, in Hume's opinion, the thesis of infinite divisibility, once that concept is properly analysed, can be seen to generate paradoxes. Because it requires us to consider *A real quantity, infinitely less than any finite quantity,* (as) *containing quantities infinitely less than itself, and so on in infinitum.* (E156) In other words, to quote Flew's paraphrase of this statement: 'it implies...that there are some finite things which are...constituted of infinite collections of other finite things, which in turn are constituted of infinite collections of infinitely smaller finite things, and so on ad infinitum.'[8]

In his elucidatory examination of Hume's views on infinite divisibility Flew draws attention to the fact that Principle B, far from being obvious, as Hume claims, incorporates two false assumptions. Firstly, it does not follow from the capacity of some thing for being divided into x number of parts that that thing exists as a compound of x number of parts, or indeed of any number of parts. Secondly, to say that some thing may be divided in infinitum is to say that there is an infinity of possible divisions and sub-divisions that can be made, and not to say that it can be divided into an infinite number of parts.[9] With the removal of these misconceptions, says Flew, the paradoxes and problems Hume finds in the concept of infinite divisibility disappear too.

Now, Flew offers no reason why Hume bases his argument on an erroneous principle. Possibly he thinks that Hume was merely muddled about the concept of divisibility in infinitum, or has swallowed without question Bayle's statements on the subject, since Kemp Smith has established that the Dictionary exerted a considerable influence on Hume's thought in this area. Certainly, a look at Bayle reveals the content of Principle B to be clearly present. In his discussion of Aristotle's response to Zeno's second objection to the existence of motion Bayle writes: '...if matter is divisible in infinitum, it actually contains an infinite number of parts, and is not therefore an infinite in power, but an infinite which really and actually exists. The continuity of parts doth not hinder their actual distinction; consequently their actual infinity doth not depend on the division; but it subsists equally in a close quantity, and in that which is called discrete.'[10]

Allowing for Hume's reading of Bayle, the question remains of why Hume should have followed Bayle's interpretation of infinite divisibility in terms of an actual rather than a potential infinity, when he does not follow, for example, Bayle's rejection of a system of mathematical points. Does Hume's adoption of Principle B signify something more than blind acceptance of Bayle's position?

A clue is provided here by the way in which Hume seeks to sidestep an objection which he cites in Section Two (T30, footnote) as having been raised against his conclusions that no finite extension is infinitely divisible. The objection turns upon a distinction of aliquot and proportional parts, asserting: *...infinite divisibility supposes only an infinite number of* *__proportional__* *not of* *__aliquot__* *parts, and (   ) an infinite number of proportional parts does not form an infinite extension.* Hume does not explain the nature of this distinction but contents himself with describing it as *entirely frivolous*, adding the rejoinder that no part of extension, be it aliquot or proportional, can be smaller than the minimal part that the imagination can conceive. Hume's omission can be made good, however, as Flew suggests,[11] by a further look at Bayle, where the distinction between the two types of parts can be seen to be related to the Aristotelian distinction between infinity conceived as potential and as actual. In Note G Bayle defines aliquot parts as: 'parts of a certain magnitude, and of the same denomination'. Such parts are contrasted with proportional parts a little earlier in Note G, when Bayle equates the distinction with one between 'parts __communicantes__, and __non communicantes__'.[12] It becomes clear from this, that aliquot parts belong to a magnitude considered as discrete, whereas proportional parts can be spoken of in the case of continuous magnitudes. Where the former type of magnitude is concerned infinite divisibility implies division into an infinite number of discrete parts; in the latter case the infinity implied is only potential, and only an infinity of possible proportional parts of the whole is envisaged, the whole itself being finite.

If Hume gave any serious thought to the distinction between aliquot and proportional parts he must have recognised that it constitutes an obstacle to accepting Principle B and that, consequently, the objection based upon it cannot be answered by an appeal to an argument in which this principle has featured as a premise. The fact that his reply takes that form would seem to indicate that the reason behind his rejection of the distinction is the same reason as accounted for his adoption of Principle B. And this reason, I propose, arises out of the atomistic type of phenomenalism advanced in the first Part of Book 1 of the Treatise. Here Hume holds simple ideas to be the ultimate elements from which knowledge of reality is to be constructed. The source of these simple ideas is said to be impressions, to which they correspond. (T4) Impressions are either simple or complex, although in the latter case they are held to be analysable into constituent simple impressions. (T2) Whereas all simple ideas are 'exact copies' of simple impressions (although Hume admits one exception at T6) and like these *admit of no distinction nor separation*, (T2) complex ideas are not limited to representing complex impressions since the imagination has the power to separate and unite simple ideas at will. (T3; T10)

Given this atomistic conception of reality as a construction out of simple impressions and ideas, Principle B becomes a necessary truth. Division of any idea by the imagination cannot be other than the analysis or separation of a complex idea into its component simple ideas. To conceive of such division as being infinite would require the preconception of an actual infinity of simple ideas. But the idea of an actual infinity is, in Hume's opinion, neither one of which experience affords us an impression nor one we can construct from simple impressions. The conclusion cannot fail to follow: *that the idea, which we form of any finite quality, is not infinitely divisible, but that by proper distinctions and separations* we may run up this idea to inferior ones, which will be perfectly simple and indivisible. (T27, central emphasis mine)

The same atomistic conception of reality also renders redundant any suggestion of a distinction between aliquot and proportional parts of a whole. For Hume, the concept of a continuous magnitude is itself ruled out by the character of his epistemology, which is why he makes the lame appeal to a conjunction of proportional parts forming no less an extension than that which would be formed by the conjunction of our minimal ideas.

What these considerations serve to reveal is the way in which the principles A and B feature in Hume's argument against infinite divisibility. Taken independently of Hume's atomism and his Principle A, Principle B does not exclude a finite extension from containing an infinite number of parts if these parts are held to be infinitesimals; hence the possibility of infinite divisibility in Hume's sense is not excluded either. This may well account for why Hume feels called upon to provide the further argument in Section Two to show: *that the idea of an infinite number of parts is individually the same idea with that of an infinite extension.* (T30); an argument taking a form which is heavily reliant upon his epistemological atomism. Principle A likewise, taken independently of B, cannot refute the infinite divisibility theorists, since they can retort that they have no need for the idea of an actual infinity anyway. Taken together, however, the two principles do rule out the possibility that the idea at the threshold of the imagination's capacity for division is an infinitesimal. To have established this is important for Hume when it comes to offering a theory of geometry based upon his particular conception of a mathematical point. But here, as indeed throughout his entire answer to infinite divisibility, Hume weights the argument in favour of his own conclusions by first reconstructing his opponent's case.

In addition to its being of a finite size, it is furthermore important for Hume's conception of a mathematical point that the minimal idea we possess of a part of extension is not an idea we must rely upon the senses to provide. This is because the senses have the defect, Hume tells us, of giving us *disproportion'd images of things, and represent*(ing) *as minute and uncompounded what is really great and compos'd of a vast number of parts.* (T28)

Thus although the senses present us with simple indivisible impressions these cannot always be taken as true representations of existing things. The impression of a mite, for example, is an impression simple and indivisible to the eye, whereas to identify that impression as the impression of a mite involves producing a complex image (notion) which applies to that simple impression. To do this, Hume informs us, *we must have a distinct idea representing every part* of a mite; and in the case of an insect *a thousand times less than a mite*, the same condition applies. Therefore it must be admitted, he thinks, despite what *common opinion* supposes, *that we can form ideas, which shall be no greater than the smallest atom of the animal spirits of an insect a thousand times less than a mite* (T28). Such ideas are provided by the imagination, and prompted by reason which informs us of the existence of objects of an imperceptible size. And rather than be surprised at our possession of ideas adequate to the minuteness of the parts of these objects, Hume thinks we should wonder at our ability to produce complex ideas of the objects themselves, considering the finite nature of that minimal image which represents each distinct part.

So when Hume, in discussing extension and geometry, tells us: *...sound reason convinces us that there are bodies vastly more minute than those, which appear to the senses; and ( ) a false reason wou'd perswade us, that there are bodies infinitely more minute;* (T48), that which reveals reason to be false in advancing to infinity cannot be minima sensibilia, for these are powerless to prevent such advance, but must be those minimal images which belong to the imagination. This means that although the minima of both the senses and the imagination, considered from the viewpoint of their respective faculties, can be regarded as true representations of the simple and indivisible nature of an atom of extension, it is the minimal image reached by the imagination when it seeks to repeatedly divide a finite idea of extension (including an idea at the theshold of sight) which must be the idea corresponding to a minimal part of extension. The distinction, present in Hume's thought if somewhat obscurely given, between the minima of the senses and of the imagination, is important in understanding his response to the infinite divisibility theorists' account of geometry. As I shall show later, by his championing a view of extension as a composition of mathematical points, Hume is led to entertain, in the Treatise, the conception of a possible geometry that exhibits the same exactness and precision as he thinks attends algebra and arithmetic. It would be misleading to speak of this as a pure geometry, since Hume regards it as descriptive of physical space. The fact that it remains an ideal rather than a real possibility is connected with the distinction above.

By the end of Section Two Hume is of the opinion that his onslaught against the infinite divisibility of extension is complete; that he has not merely presented its advocates with certain difficulties to be overcome, but has succeeded in producing a counter-demonstration. (T31) His closing remarks in this section show that he sees his triumph as lying in the vindication of mathematical points, and consequent delivery of geometry from the paradoxes of infinite divisibility. (T32/33) But the problem of accommodating within an empiricist framework a system of geometry based on mathematical points, together with the problem of reconciling the concept of a mathematical point with his empiricist theory of meaning, land Hume in considerable difficulty which is never satisfactorily resolved in the Treatise and leads to certain modifications in the Enquiry.

Before proceeding to consider Hume's account of the genesis of our idea of space, brief mention of an argument to be found at T30 is in order. This argument, which Hume holds in high favour and attributes to a Mons. Malezieu, asserts: *'Tis evident, that existence in itself belongs* *only to unity, and is never applicable to number, but on account of the unites, of which the number is compos'd.* Hume employs it to construct a dilemma for those who hold extension to be both infinitely divisible and determinable quantitatively: either extension, as a measurable quality, exists as a composition of units (indivisible parts), or it does not exist at all. Since Hume provides no reason why it should be accepted that: *...the unity, which can exist alone, and whose existence is necessary to that of all number,...must be perfectly indivisible, and incapable of being resolved into any lesser unity* (T31), the dilemma has little force. The passage is of some interest, however, for the light it throws on Hume's view of arithmetic. Hume, it appears, wholly accepts the conception of numbers as composites of units. This suggests that he must regard whole numbers as complex abstract ideas based upon the simple idea of a unit, which accounts for our being said to possess *a precise standard...of equality and proportion* in arithmetic, namely: *When two numbers are so combin'd, as that the one has always an unite answering to every unite of the other, we pronounce them equal.* (T71) We are never told on what authority we employ the concept of a unit, but it is difficult to see how it can be other than an abstract idea given Hume's view of number, even if he does follow Locke and hold that the idea of unity is furnished by all our perceptions. Indeed, arithmetical ideas seem to be placed in a class of their own by Hume, as ideas which require no image in order to be 'before the mind'. Thus one finds Hume contrasting the minimal image the imagination can produce with our ideas of rational fractions (T27), and admitting that we have no *adequate idea* of large numbers, *but only a power of producing such an idea, by* (the) *adequate idea of the decimals, under which the number is comprehended.* (T23) It therefore appears to be the case, that infinity, considered as a mathematical idea, is ruled out by Hume not because of the absence of an image corresponding to it, but because of our inability to construct it mathematically. As I have said earlier, the concept of an actual infinity is, in Hume's opinion, one which neither experience nor mathematics can provide.

2

Empiricist concerns are foremost in Hume's mind when he sets out to consider in Section Three, *the other qualities of our ideas of space and time.* The major part of this discussion is devoted to his account of the genesis of our idea of time, but commentators have often taken what is said of time as applying also to space. Certain of Hume's remarks are indeed suggestive of an analogy holding between the two cases: T35, for example: *As 'tis from the disposition of visible and tangible objects we receive the idea of space, so from the succession of ideas and impressions we form the idea of time.* Likewise T39/40: *The ideas of space and time are...no separate or distinct ideas, but merely those of the manner or order, in which objects exist.* The question at issue, therefore, is the nature and extent of this analogy. To determine this let us first look at what Hume says about the origin of our idea of time.

Hume informs us:

> *The idea of time, being deriv'd from the succession of our perceptions of every kind, ideas as well as impressions, and impressions of reflection as well as of sensation, will afford us an instance of an abstract idea, which comprehends a still greater variety than that of space, and yet is represented in the fancy by some particular individual idea of a determinate quantity and quality.* (T34/35)

Now, it is plain enough, that if the idea of time is derived from the experienced succession of perceptions, and from this alone (T35), then it does not arise from a particular impression. Hume makes explicit mention of this at T36, and claims it *arises altogether from the manner, in which impressions appear to the mind, without making one of their number.*

He then proceeds to identify time (duration) with the experience of succession or change: ...*since it appears not as any primary distinct impression,* (it) *can plainly be nothing but different ideas, or impressions, or objects dispos'd in a certain manner, that is, succeeding each other* (T37), an identification which is responsible for his remarkable assertion that we can apply the idea of duration to unchanging objects only by means of a *fiction.*

Kemp Smith, bearing in mind Hume's assertion that space and time are both abstract ideas, interprets Hume's views on the genesis of the idea of space as running parallel to the above account of time with respect to the absence of a particular impression from which the idea arises. To grasp Kemp Smith's position it is necessary to quote it fairly fully:

> Space...is the name proper to a certain specific manner or order of arrangement of visual and tactual impressions; time is the name proper to that other manner or order of arrangement,...which holds in the case of all our perceptions.... The 'manner' of arrangement, as being an arrangement of the simple perceptions, is not given in the content of any one perception, and also does not consist in any mere summation of them. The arrangement is over and above the perceptions.... It is "the manner of their appearance". They are, Hume is virtually saying, contemplated or intuited...but not sensed. They are non-impressional.[13]

In this section I shall argue against Kemp Smith, that the 'arrangement' concerned in the case of spatial ideas does lie 'in the content' of spatial perceptions in Hume's opinion, and that, therefore, the genesis of the abstract idea of space is impressional in a way which serves to distinguish it from that of time. Apart from their being good internal reasons why Hume would have avoided wherever possible (time, presumably, presenting an impossibility), an account such as Kemp Smith attributes to him, I do not think that Section Three warrants it, or that it is consistent with remarks he makes elsewhere about extension.

Section Three begins with Hume reminding his reader that *...every idea, with which the imagination is furnish'd, first makes its appearance in a correspondent impression.* (T33) Armed with this principle he sets out to investigate the origin of our ideas of space and time. The idea of extension, we are told, is acquired in the following manner: *Upon opening my eyes, and turning them to the surrounding objects, I perceive many visible bodies; and upon shutting them again, and considering the distance betwixt these bodies, I acquire the idea of extension.* (Ibid) This statement might well seem to suggest that Hume thinks that the idea of extension is connected genetically with an image of darkness, but it is abundantly clear from the later discussion of vacua that this is not his opinion. For in Section Five Hume draws a distinction between 'true distance' and 'invisible distance', and maintains that the resemblance borne by the effects of the latter upon our senses to the effects of the former is responsible for our false supposition that we have an idea of empty space. (T59/60) True distance alone is said to be capable of conveying the idea of extension; and true distance is visible distance, that is, distance itself filled with objects. Hume thus denies that the phenomenon of darkness can give rise to any idea of extension on the ground that darkness *is no positive idea, but merely the negation of light, or more properly speaking, of colour'd and visible objects.* (T55) And even where darkness is interspersed with visible objects it cannot provide an impression for the idea of extension without matter, says Hume, since the invisible distance is still *nothing but darkness,...without parts, without composition, invariable and invisible.* (T57) So it is evident enough that Hume does not think that the idea of extension has anything to do with the phenomenon of darkness.

It is worthy of note, however, that Hume's reason here is not, as Flew suggests (p.469), based on the fact that a black visual-image field, as an image, cannot represent a vacuum, which is an idea of nothingness; it rather points to the undifferentiated nature of a black visual-image field as being incapable of providing an image to which our idea of space as a composite corresponds. Hume's rejection of the possibility of vacua follows not so much from the absence of any possible impressional source for the idea as from his argued conclusion that: *The parts, into which the ideas of space and time resolve themselves, become at last indivisible; and these indivisible parts, being nothing in themselves, are inconceivable when not fill'd with something real and existent.* (T39; Cf.T53)

In view of the above, it would seem that the *distance* Hume is referring to at T33 must be what he calls 'true distance'. This helps to explain how Hume thinks he can move from talking about the idea of extension as being connected with the consideration of distance between objects, to the assertion: *The table before me is alone sufficient by its view to give me the idea of extension.* (T34) Any extended object would fill this role where Hume is concerned. What is startling, however, is the description Hume gives to the perception of the table: *...my senses convey to me only the impressions of colour'd points; dispos'd in a certain manner.* To assert that we perceive material objects as composites of coloured points is, to say the least, a very curious claim, and one for which Hume's phenomenalist atomism must be held responsible, since it is not demanded by his rejection of infinite divisibility. His case against the latter was constructed on the basis of Principles A and B: the conclusion that we conceive extension as a composition of simple indivisible parts was held to follow from these alone, and there was no appeal to an impression on which this character of our idea of extension depends. But Hume, carried away on a wave of his particular brand of empiricism, obviously feels constrained to point to just such an impression in Section Three. So, after reaffirming the principle of significance advanced in Part One: *...every idea* *is deriv'd from some impression, which is exactly similar to it* (T33), he proceeds to describe an impression of an extended object in such a way as to make possible the conclusion *that the idea of extension is nothing but a copy of these colour'd points, and of the manner of their appearance.* (T34)

What is, however, important for Hume from the point of view of the answer he proposes to make to the infinite divisibility theorists' objection that mathematical points are non-entities and consequently cannot *form a real existence* by their conjunction (T40), is that impressions of coloured and/or tangible objects are proven to be the only impressions from which our idea of extension can originate. For this, he thinks, enables him to reply to the question: *What is our idea of a simple and indivisible point?*, that we can only conceive of such a point as being either coloured or tangible: without these sensible qualities it is *utterly annihilated to the thought or imagination.* (T38/39) Thus, although Hume concedes that the idea of space is an abstract idea and therefore not an exact copy of any single impression, he allows only that it abstracts from *the peculiarities of colour* characterising particular impressions, and not that it abstracts from the quality of colour itself. Or if the latter is possible (and Hume's remarks are ambiguous here) tangibility must take the place of colour. (T34) The idea of space is abstract because it is formed on the basis of an observed 'resemblance' among impressions of extended objects, this resemblance covering both impressions of sight and touch and consisting in *that disposition of points* (or parts), or manner of appearance, in which they agree (T34). It is difficult to be certain of what Hume has in mind here, and I can only think that the resemblance said to exist among different configurations of points must refer to spatial dimensionality: the coloured/tangible points are arranged in patterns of one, two or three dimensions. The abstract idea of space will therefore be the idea of a three-dimensional system of coexistent coloured or tangible points, and Hume's claim that *We have ( ) no idea of space or extension, but when we regard it as an object either of our sight or feeling* (T39), will seem safe enough to him.

The assertion that the abstract conception of space cannot dispense with certain sensible qualities paves Hume's way to arguing for a possible exact mathematics of space, based upon a spatial unit which is simple, indivisible and coloured or tangible. It is this conception of a mathematical point that Hume triumphantly produces as the cornerstone of his case against infinite divisibility. He stresses the difference between a mathematical point and a physical point in this context, and in a way which connects with the earlier mentioned distinction between the minima of the senses and of the imagination. A physical point is, he says, a real extension, that is, composed of parts; as such, it is divisible by the imagination. A mathematical point, on the other hand, represents the idea at the limit of the imagination's divisional capacities, and to *bestow( ) a colour or solidity on these points* furnishes them with some sort of reality, albeit it seems only an ideal one. (T40) To have achieved this objective, however, Hume need have made reference solely to the manner of arrangement of the coloured/tangible <u>parts</u>, where impressions of extended objects are concerned: it is a confused assessment of his case brought about by his underlying atomism which prompts the insistence on points at T34; at other places in the <u>Treatise</u> where the impression of extension is discussed reference is almost exclusively to parts.

It is the considerations described above which underlie Hume's claim that the idea of space, like that of time, is not a separate idea but represents only *the manner or order, in which objects exist* (T40). This remark tends to conceal the difference between the ideas of space and time which Kemp Smith overlooks. For whereas in the case of time the 'manner' is not directly represented in the content of any individual impressions, it is otherwise with space. Hume makes clear his opinion that our abstract idea of space has its origin in resembling compound impressions. Certainly, the mind has to take note of the resemblance, just as in the case of time it takes note of the resemblance among different series of changing perceptions. The ground of the latter resemblance, however, is the subject's mode of perceptual awareness: the ground of the former is the compound impressions he experiences through sight and touch. (T38)

Other passages in the Treatise support the view that Hume takes the relational content of the idea of space to be much more directly impressional in its origin than is true of time; and that, consequently, his explanation of the genesis of our idea of space fits more easily into his theory of knowledge. For if Kemp Smith is right to claim that the idea of extension involves for Hume an ordering which is additional and external to perceptions themselves, then it will follow that Hume holds none of our perceptions to be extended. Yet in discussing the immateriality of the soul and commenting on the opinion of some that an extended or material soul cannot be conjoined with simple indivisible perceptions, Hume declares that all our perceptions other than those of sight and touch exist nowhere: a perception which exists nowhere being one the parts of which *are not so situated with respect to each other, as to form any figure or quantity*. (T235/236) Were this not so, he says, we would have to allow that perceptions other than those of sight and touch could give rise to the idea of extension, whereas it is the case that only *what is colour'd or tangible...has parts dispos'd after such a manner, as to convey that idea*. (T235) In other words, Hume holds that perceptions of sight and touch (although not all of the latter Cf.T230) are extended. This is confirmed a little later on:

> *That table, which just now appears to me, is only a perception, and all its qualities are qualities of a perception. Now the most obvious of all its qualities is extension. The perception consists of parts....so situated, as to afford us the notion of distance and contiguity; of length, breadth, and thickness.* (T239)

And any remaining doubt as to Hume's position is dispelled by his concluding comment: *...the very idea of extension is copy'd from nothing but an impression, and consequently must perfectly agree to it. To say the idea of extension agrees to any thing, is to say it is extended.* In Book Two of the Treatise also, we find the remark: *...any one may easily observe, that space or extension consists of a number of co-existent parts dispos'd in a certain order, and capable of being at once present to the sight and feeling* (T429), a statement which supports the existence of a close relationship between the complex relational nature of our abstract idea of space and the complexity which characterises our impressions of extension.

All in all then, neither Section Three itself requires, nor do the additional passages quoted give strength to, Kemp Smith's claim that the complex idea of space is 'non-impressional' and that Hume's 'account of the idea of space, though more obscurely and ambiguously stated, runs parallel to his account of the idea of time'. (op.cit.p.276) While it is true that Hume suggests some parallelism (and in view of the previous history of philosophy it would be surprising had he not done so), this should not be allowed to obscure a distinction which is important in relation to his empiricism. For although it holds of both space and time, considered as abstract ideas, that they do not refer to one particular impression, it is true of time alone that its genesis is independent of the content of all particular impressions.

3

Hume's primary motive for discussing space and time in the Treatise, and that which gave direction to this discussion was his concern with mathematical knowledge, especially with geometry. It is to his opinions on this subject that I now turn. The task of interpretation here is made difficult, however, by the paucity of detailed comment from Hume. Section Four of Hume's exposition of space and time, together with the first section of Part Three of the

Treatise Bk. 1, comprises the only material from which a construction of his views on mathematics can be attempted. Since the Enquiry contains even less comment it is not surprising to find some divergence of interpretation among commentators.

Section Four commences with Hume presenting a rebuttal of objections raised against a system of indivisible units of extension. When he comes to those drawn from mathematics he maintains that his quarrel is not with the definitions provided by mathematicians for concepts central to geometry, but with the demonstrations put forward. For while they define a surface as *length and breadth without depth*, a line as *length without breadth or depth*, and a point as *what has neither length, breadth or depth*, these mathematicians refuse to allow that extension is composed of indivisible points or atoms; and yet, says Hume, *'Tis evident that all this is perfectly unintelligible upon any other supposition.*(T42) He backs up this last claim with two attacks on theories to the contrary using weapons from his own forge, before concluding that *the definitions of mathematics destroy the pretended demonstrations.* (T44) And to make certain of victory he casts a final blow:

> *But I go farther, and maintain, that none of these demonstrations can have sufficient weight to establish such a principle, as this of infinite divisibility; and that because with regard to such minute objects, they are not properly demonstrations, being built on ideas, which are not exact, and maxims, which are not precisely true. When geometry decides anything concerning the proportions of quantity, we ought not to look for the utmost precision and exactness. None of its proofs extend so far. It takes the dimensions and proportions of figures justly; but roughly, and with some liberty. Its errors are never considerable; nor wou'd it err at all, did it not aspire to such an absolute perfection.* (T44/45)

Here we have the first of Hume's statements to the effect that geometry is not, unlike arithmetic and algebra, an exact and precise science. Later he marks the distinction by speaking of geometry as an *art* rather than a *science.* (T70) Its defect is said to lie in the absence of a true and precise standard of equality by which to measure figures and determine their true proportions. We have to make judgments solely on the ground of how they appear to us. Thus lack of an accurate standard, Hume thinks, affects geometry irrespective of whether extension is taken as infinitely divisible or as finitely divisible only. For although geometry as the science of 'atomistic' space is in theory capable of being precise, the proportions and equality of figures being determinable by computing the numbers of mathematical points they contain, this standard is in practice, says Hume,

> *entirely useless....For ( ) the points, which enter into the compositon of any line or surface, whether perceiv'd by the sight or touch, are so minute and so confounded with each other, that 'tis utterly impossible for the mind to compute their number...* (T45)

Thus in geometry, he thinks, we have no choice but to utilise a sensible standard of equality, one which cannot go beyond appearance. For although any initial rough judgment can be made more refined by the use of standard measures, these measures themselves depend upon the sense for their construction and employment.

It is clear therefore, that when Hume speaks of practical geometry as being an inexact mathematics of space, he is contrasting it with the theoretical possibility of an exact spatial arithmetic based on the concept of a mathematical point as a unit and employing definitions of its basic concepts. Where practical geometry is concerned, its concepts and axioms are *deriv'd merely from appearances.* (T71; Cf.T50-52) Practical geometry is to be denied scientific status only when it is viewed against the background of a purely theoretical and ideal conception of geometry as exactly descriptive of physical space, where the nature of this space is only incompletely accessible to the senses. Such a theoretical geometry is for Hume an ideal which the imagination and reason, working together, construct on the basis of the imagination's minimal idea of a part of extension, motivated by reason's conviction that *nature is susceptible*...(of) *prodigious minuteness* beyond that which the senses can perceive. (T71; T48; T28) Thus although this ideal geometry possesses a precise standard of equality, this standard is, says Hume, *plainly imaginary*. However, Hume is forgetful of his own position when he urges further that since the only practical standard of equality available to us is a sensible one, *the notion of any correction beyond what we have instruments and art to make, is a mere fiction of the mind, and useless as well as incomprehensible*. (T48) Imaginary it may well be called on the grounds that the imagination and not the senses is both its source and field of employment; but *a mere fiction of the mind* and *incomprehensible* it cannot be, if Hume's description of practical geometry as an *imprecise* science is to remain meaningful and his earlier arguments in support of a theory of mathematical points are to be seen as making any kind of bulwark against scepticism in geometry.

The fact of the matter is that in the Treatise Hume answers the theoretical case for the infinite divisibility of extension with an equally theoretical conception of extension as a composition of discrete indivisible points or atoms. He takes this as a true representation of the character of physical space, of which geometry provides the description. Consequently, he must allow that the 'imaginary' standard of equality has at least theoretical application to physical and mathematical space. For him to deny this on the grounds of its inapplicability to sensible or perceptual space amounts to the destruction of his entire case against infinite divisibility, and also renders incomprehensible his remarks on the limitations of practical geometry. Whatever may be the position in the Enquiry, in the Treatise Hume cannot disregard the contrast between a theoretical geometry held to be exactly descriptive of physical space, and a practical geometry which holds good for perceptual (sensible) space but which may remain only inexactly descriptive of the properties of physical space. And it is with this contrast in mind that Hume's attribution of imperfection to geometry at T45 and T71 must be understood. For, states Hume, although the basic concepts and first principles of (practical) geometry are all taken from sensible experience, they *depend on the easiest and least deceitful appearances*, and therefore *bestow on their consequences a degree of exactness, of which these consequences are singly incapable.* (T72) Thus Hume feels no hesitation about classing this geometry alongside arithmetic and algebra as an object *of knowledge and certainty*, when he comes to granting this title to certain *philosophical relations* of ideas. (T69/70)

Hume's division of the *seven different kinds of philosophical relation* into two categories, *such as depend entirely on the ideas, which we compare together, and such as may be chang'd without any change in the ideas* (T69), is a distinction concerning the epistemological status of different relations. Only constant relations, those which are invariable given no change in the relata, are held to qualify as objects of knowledge; inconstant relations yield only probable conclusions. In as far as the division corresponds to any epistemological distinction currently recognised, it would seem to differentiate truths which are in some sense necessary from those which are contingent. From Hume's inclusion of geometry within the former category it emerges that he considers some type of necessity attends geometrical axioms and inferences. Just what kind of necessity this is remains unclear, bearing in mind his assertion that the first principles of this science are drawn from the senses.

Hume offers little assistance here, although some clue can be taken from what he says about two of the three non-mathematical relations he holds to be objects of knowledge, namely, resemblance and degree in quality. These relations are, he thinks, *discoverable at first sight, and fall more properly under the province of intuition than demonstration* (T70) whereas the discovery of mathematical relations involves, in the main, *a chain of reasoning* (T71). It is by virtue of the last mentioned character that mathematics constitutes for Hume the only true science, but it must not be assumed from his statement that he thinks intuitive or immediate inferences have no place in mathematics. His point is only that deductive reasoning is a feature of mathematics alone: and this tells us nothing about the logical status of the premises employed, or indeed of those propositions involved in the chain of inference. Since the axioms of geometry are said to be provided by the senses, any necessity belonging to them cannot be conceptual or analytic even in a broad sense; it must be synthetic; and the synthetic relation must itself be based upon an intuitive comparison of certain sensible ideas. Hume seems in some doubt when speaking of the relation of resemblance as to whether the act of comparison is to be attributed to the senses or to the intellect. But his more considered opinion favours the latter, as indeed it must if he is to hold, as at T166, that the necessity present in an invariable relation of ideas lies in the act of understanding by which we consider and compare the ideas.

These considerations suggest that Hume thinks that the nature or properties of certain perceptions determine the mind to find inseparable connections among them. That as well as being true where objects resemble one another, or admit of comparison in terms of the degree possessed of some quality, it is also true of certain spatial properties of objects and figures. These properties, as perceived, determine the mind in its conception of the relations holding between them, and render the non-existence of those relations inconceivable, given that the nature of perceptual (sensible) space remains unchanged. The necessity which belongs to geomatrical axioms and to the body of geometric truths as a whole will therefore ultimately reside in the constant character of perceptual space; it is a synthetic necessity. Confirmation of this viewpoint is given by Hume when he rejects the suggestion that a straight line can be defined as the shortest distance between two points, and maintains that this 'definition' is really a property of a straight line which has to be discovered. (T50) Likewise, when he writes: *'Tis from the idea of a triangle, that we discover the relation of equality, which its three angles bear to two right ones; and this relation is invariable, as long as our idea remains the same* (T69), he is asserting that this relation of equality is a spatial property of triangular objects, whether given or constructed: it is a property we discover from examining the triangle, not a definition of a geometrical concept.

Geometrical propositions, then, are not conceptual truths, in Hume's opinion. Rather, they embody descriptions of the properties of perceptual space. They are necessary truths but not analytic. In this regard Hume approaches the position later assumed by Kant, namely that mathematics is a body of necessary synthetic truth. But the resemblance is limited. While it is true that Hume treats the propositions of geometry differently from the principle of the uniformity of nature, for example, or the causal principle, both of which are held to be contingent propositions which can be denied without contradiction (T89; T95), geometry remains all along an empirical a posteriori science. Hume makes no attempt to secure its necessary status with the Kantian formula. Thus, although he maintains that where the propositions of geometry are concerned: *...the person, who assents, not only conceives the ideas according to the proposition, but is necessarily determin'd to conceive them in that particular manner, either immediately or by the interposition of other ideas* (T95), the only factor he makes appeal to as 'determining' is the character of sensible space, the invariant nature of which seems to rest, in the Treatise, upon the constant character of physical space. In view of the scepticism voiced by Hume elsewhere in the Treatise about our knowledge of empirical reality and our belief in its constancy, it must be concluded that he fails in his objective: to account for and secure the certainty of geometrical knowledge without relinquishing his belief in its empirical status.

Turning to the Enquiry there is little to indicate any shift in Hume's opinion of geometry as a body of synthetic but necessary knowledge. No discussion of the genesis of our ideas of space and time is present, and although Hume retains his Treatise antagonism to infinite divisibility he does not repeat any of his earlier arguments against the thesis nor those in support of a system of mathematical points. Indeed, in a footnote to his brief discussion of infinite divisibility at E156, he makes a statement that suggests he has moved away from the Treatise conception of a mathematical point as a theoretical unit of extension and distinct from a physical point. In the earlier work the conception of extension as a composition of physical points was rejected on the grounds that a physical point can be divided by the imagination; whereas here, a physical point is said to represent that part of extension *which cannot be divided or lessened, either by the eye or imagination.* Also, and significantly, there is no reference in the Enquiry to the absence of an exact standard of equality in geometry. It is classed as a science alongside arithmetic and algebra. (E25)

Some commentators have taken this last mentioned feature of the Enquiry as marking a change from the Treatise view of geometry as an empirical science, finding support for their interpretation of the Enquiry position in Hume's obscure comment that intuitively or demonstratively certain propositions

> *are discoverable by the mere operation of thought, without dependence on what is anywhere existent in the universe. Though there never were a circle or triangle in nature, the truths demonstrated by Euclid would for ever retain their certainty and evidence.* (E25)

Flew thinks that in the Enquiry Hume is to be seen as 'restoring pure geometry to its place alongside the other two elements of the trinity'[14], although he does not commit himself to the view that Hume regards mathematics as wholly analytic, unlike Noxon who holds that in the Enquiry Hume takes all three mathematics 'to alike express analytic truths which rest upon the law of non-contradiction'[15]. A third commentator, R. F. Atkinson,[16] finds no significant difference between the earlier and the later work and thinks that insofar as Hume's remarks on mathematical propositions permit the application of the synthetic/analytic distinction, mathematics falls within the province of synthetic necessary, rather than analytic, truth. Where geometry is concerned I follow Atkinson's opinion in finding no reason to suppose that Hume abandoned the Treatise stand. I suggest that the lack of any reference to the imprecision of geometry compared with arithmetic and algebra reflects Hume's relinquishment of the Treatise conception of a possible ideal precise geometry with which practical geometry stands contrasted. There is no suggestion present anywhere in the Enquiry that Hume wishes to retract the earlier view that the basic concepts and principles of geometry are all derived from appearances, and the statement quoted from E25 is far from being conclusive evidence of a change in Hume's opinion of the epistemological status of geometrical propositions. It can be interpreted so as to remove any apparent inconsistency with the claim that the impossibility of conceiving the negation of a geometrical axiom has the same ground in the Enquiry as in the Treatise, namely, the invariant character of sensible space. For although Hume holds that the basic ideas and principles of geometry are taken from the sensible world, this in no way commits him to holding all geometrical concepts to be exemplified in nature. The geometrician can construct further concepts as figures in perceptual space, utilising those primary concepts which are empirically given. The properties of the constructed figure could then be determined by what Hume calls 'the mere operation of thought'. This would involve the inspection and measurement of the figure to discover those relations of its parts which are immediately given, then the determination of further of its properties from these by reasoning. Such constructs would represent possible forms of natural objects even if no natural examples of them exist, and would serve to furnish information about the properties of sensible (perceptual) space. Certainly, on this interpretation, the way in which thought can operate alone to produce geometrical knowledge will differ from its activity in determining arithmetical and algebraic truths, but there is no reason why we should suppose Hume intended the process of thought involved to be the same in both cases. Indeed, insofar as the brevity of his comment on arithmetic permits any tentative judgment of his opinion on the logical status of its propositions, the nature of the necessity involved here appears to be conceptual or analytic in a wide sense, and not synthetic as in geometry.

* * * * * * * * * *

In this paper I have argued for the existence of a close relationship between Hume's approach to geometry and his concern with refuting the doctrine of infinite divisibility. This preoccupation led, in the <u>Treatise</u>, to the conception of an ideal theoretical geometry of physical space, based upon the idea of a mathematical point. Because of the limitations of our senses, this remained an ideal in contrast with which a full scientific status was withheld from the geometry of sensible or perceptual space, although the latter was regarded as producing a body of necessary synthetic knowledge. In the <u>Enquiry</u> the notion of an ideal geometry was abandoned, probably because Hume came to realise that his treatment of infinite divisibility in the Treatise constituted an uneasy marriage of empiricism and rationalism. The necessary synthetic status of geometry remains unchanged, although here, as in the earlier work, Hume fails to provide any adequate grounds for the type of necessity held to be involved, and thus fails in his objective to safeguard the certainty of this branch of mathematical knowledge.

Rosemary Newman
University of York

1. J. Noxon, Hume's Philosophical Development (Oxford, 1973), 115.

2. A. Flew, 'Infinite Divisibility in Hume's Treatise', Rivista Critica di Storia Filosofia, 22, (1967), 457.

3. J. Y. T. Greig, ed., The Letters of David Hume (Oxford, 1932) Vol. 2, No. 465, p.235.

4. D. Hume, Abstract, ed. J. M. Keynes & P. Sraffa, (Archon Books, Connecticut, 1965) 26/27.
   Enquiry, ed. L. A. Selby-Bigge, (Oxford, 1902), 156. All references to the Enquiry will be made to this edition, and all references to the Treatise will be made to the Selby-Bigge edition (Oxford, 1946).

5. J. Locke, An Essay Concerning Human Understanding, Bk.2, Chapter 17, Section 12.

6. Ibid., Chapter 16, Section 8.

7. Ibid., Chapter 17, Section 8.

8. Flew, op.cit. 460.

9. Ibid.

10. Des Maizeaux ed. Mr. Bayle's Historical & Critical Dictionary (London, 1738, 2nd. Edtn.) Vol. 5, 609.

11. Flew, op.cit. 465.

12. *The Oxford English Dictionary*, Vol.2, 699, bears the following entry: communicant (adj.): sharing; participating; having a part in common. 1870 Bowen. *Logic* v.115; Two communicant or overlapping parts.

13. N. Kemp Smith, *The Philosophy of David Hume* (London & New York, 1941), 274.

14. *Loc.cit.* above, ref. 1.

15. A. Flew, *Hume's Philosophy of Belief* (London, 1961) 62.

16. R. F. Atkinson, 'Hume on Mathematics' *Philosophical Quarterly*, 10, (1960) 127-139.