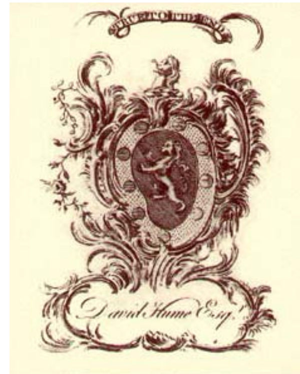

Hume's Probability Argument of I, iv, 1
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HUME'S PROBABILITY ARGUMENT OF I,iv,1

In the *Treatise*, I,iv,1, Hume presents an argument which, in the barest of outlines, goes as follows:[1]

(P1) Every proposition has a probability less than one.

(P2) If reason were the basis of our beliefs, then we would have no beliefs. (follows from (P1))

(P3) We in fact do have beliefs.

Hence,

(P4) Reason is not the basis of our beliefs.

The argument has not been particularly well received. D.C. Stove, for example, refers to it as being "not merely defective, but one of the worst arguments ever to impose itself on a man of genius."[2] While not everyone is as unsympathetic as Stove, it nonetheless is difficult to find commentators favorably disposed toward the argument.[3]

Various sections of the *Treatise* are notoriously unclear. I,iv,1 is such a section; as such, it is difficult to say just what Hume intended his argument to be. My contention in the present paper is that there are two reasonable ways of reconstructing Hume's argument, both consistent with what Hume writes in I,iv,1. The first reconstruction is clearly unsound; the second reconstruction fares somewhat better. In particular, my contention will be that this second reconstruction, if not sound, is at least valid and contains no obviously false premises.[4] Allow me to make some preliminary comments before presenting these reconstructions.

The reconstructions to follow will primarily be concerned with premise (P2) above; in particular, the reconstructions will focus on how Hume can claim that (P2) follows from (P1). Premise (P1) is a common sceptical claim, not obviously false,[5] and certainly not unique to Hume. As such, it strikes me that the more interesting part of the argument is Hume's claim that (P2) follows from (P1), and such will be the focus of the reconstructions (although (P1) will be discussed toward the end of this paper).

Concerning (P2), why does Hume think this follows from (P1)? His reasoning is contained in the following passage:

> Having thus found in every probability, beside the original uncertainty inherent in the subject, a new uncertainty deriv'd from the weakness of that faculty, which judges, and having adjusted these two together, we are oblig'd by our reason to add a new doubt deriv'd from the possibility of error in the estimation we make of the truth and fidelity of our faculties. This is a doubt, which immediately occurs to us, and of which, if we wou'd closely pursue our reason, we cannot avoid giving a decision. But this decision, tho' it shou'd be favourable to our preceeding judgment, being founded only on probability, must weaken still further our first evidence, and must itself be weaken'd by a fourth doubt of the same kind, and so on in infinitum; till at last there remain nothing of the original probability, however great we may suppose it to have been, and however small the diminution by every new uncertainty. No finite object can subsist under a decrease repeated in infinitum; and even the vastest quantity, which can enter into human imagination, must in this manner be reduc'd to nothing. (T182)

Not for nothing did I claim that this section of the Treatise is unclear. At best, I can discern only the outline of Hume's reasoning, which seems to go as follows: first, when we assign a probability to some proposition, reason dictates that we re-evaluate this probability. In particular, reason dictates that we "add a new doubt deriv'd from the possibility of error in the estimation we make of the truth and fidelity of our faculties." Note that Hume never specifies the object of this 'new doubt.' He might mean a) that our doubt concerns whether the probability we assigned the proposition is correct, or b) that our doubt concerns the evidence on which we based the assignment of that probability.

Whatever the case, this 'new doubt' must "weaken still further our first evidence," presumably resulting in a lowering of the probability we originally assigned to the proposition. We then repeat this doubting process, adding new doubts of the same kind in infinitum, "till at last there remain nothing of the original probability."

In short, then, reason dictates that we continually re-evaluate the probability we assign a proposition, such that each re-evaluation lowers the probability of that proposition. This process, Hume seems to think, will drive the probability of any proposition to zero (or more accurately, the probability will approach a limit of zero), and hence we would have no justification for believing that proposition.

Allow me to offer two reconstructions, both consistent with this outline of Hume's argument, but which are somewhat more specific as to why Hume might think the probability of any proposition, based on reason, will be driven to a limit of zero.

Reconstruction 1: Let Q be any proposition. For the sake of convenience, let us suppose that Q is a proposition which I would ordinarily believe to have a high degree of probability, such as "the sun will rise tomorrow." Suppose, for the sake of convenience, that my original estimation of the probability of Q is .99. That is, my original estimation is

1) $P(Q) = .99$

Realizing that I am a fallible reasoner, I agree with Hume that "in every judgment, which we can form concerning probability ... we ought always to correct the first judgment...." (T 181-182). That is, I realize that I may have wrongly estimated the probability of Q, and that the probability of Q may in fact not be .99. Suppose I estimate that the probability of my having assigned the correct probability to Q is .99. That is,

2) $P(\, P(Q) = .99) = .99$

And following the same line of reasoning, I realize that this last assignment of probability has only a .99 probability of being correct; that is,

3) $P(\, P(\, P(Q) = .99) = .99) = .99$

And so on indefinitely. Referring back to the passage quoted before the beginning of this reconstruction, we find that this reconstruction is consistent with this passage. We did indeed find in every probability a new uncertainty, that uncertainty being the possibility of having wrongly estimated the preceding probability. And we did, consistent with the passage, "add a new doubt deriv'd from the possibility of error in the estimation we make of the truth and fidelity of our faculties."

Unfortunately, the argument is fallacious insofar as Hume claims that the argument will drive the probability of Q to (a limit of) zero. It is fallacious because a higher order probability, such as $P(\, P(Q) = .99)$, will not, under any ordinary probability theory, affect the lower order probability $P(Q) = .99$. So for example, in line (3), where $P(\, P(\, P(Q) = .99) = .99) = .99$, the probability of Q remains .99. And hence Hume's claim that each new probability weakens the first, "till at last there remain nothing of the original probability," is false.

Reconstruction 2: There is, however, an alternative reconstruction of the argument of I,iv,1. Let Q be any proposition. Let E1 represent my total body of evidence supporting Q. That is, if someone were to ask me what justification I have for believing Q, I would present E1 as my total body of evidence for that belief.

A brief note is in order here concerning the notation to be used. The concern in this reconstruction is with the probability of Q based on evidence E1; as such, the following notation will be used:

$$P_{E1}(Q)$$

to be read as "the probability of Q on evidence E1," where E1 is understood to be my total body of evidence supporting Q.

The probability of Q depends, of course, on a) the degree of support which E1 gives to Q, and b) the probability of E1. In particular, the probability of Q equals the support given by E1 multiplied by the probability of E1, i.e.,

1) $P(Q) = P_{E1}(Q) \quad P(E1)$

I suspect the reader will find that something seems rather odd about speaking of probability in this way. I think the reason this sounds odd is that, when we ordinarily use the term 'evidence', we are referring to something which is either a) true, or b) something we are fully justified in believing. That is, 'evidence' generally refers to something which has a probability of one.[6] And if the evidence has a probability of one, then it is unnecessary to include the consideration of P(E1) in line (1), since it will not affect the calculation of P(Q).

But we need to keep in mind premiss (P1), that every proposition has a probability less than one. Since E1 would presumably consist of a set of one or more propositions, we know that E1 will have a probability less than one. As such, P(E1) will affect P(Q), and hence it is only reasonable to include it in the consideration of P(Q).

So given that E1 has a probability less than one, and given that P(E1) will affect our calculation of P(Q), we must now consider the probability of E1. Let E2 be the total body of evidence in support of E1, such that if someone were to ask me what justification I have for believing E1, I would present E2 as my total body of evidence for that belief. That is,

2) $P(E1) = P_{E2}(E1) \cdot P(E2)$

Letting En represent my total body of evidence in support of En-1, and repeating the reasoning, we have

3) $P(E2) = P_{E3}(E2) \cdot P(E3)$

4) $P(E3) = P_{E4}(E3) \cdot P(E4)$

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.
.

n) $P(En-1) = P_{En}(En-1) \cdot P(En)$

Turning our attention back to line (1), by repeated substitutions we arrive at

$$P(Q) = P_{E1}(Q) \cdot P_{E2}(E1) \cdot P_{E3}(E2) \cdot P_{E4}(E3) \cdot \ldots \cdot P_{En}(En-1) \cdot P(En)$$

It should be noted that this reconstruction is also consistent with the passage quoted before the first reconstruction.[7] As Hume claims, we did find in every probability a new uncertainty. And because of this uncertainty, we must "add a new doubt deriv'd from the possibility of error in the estimation we make of the truth and fidelity of our faculties." Here, the "possibility of error" must be taken to mean that no body of evidence will fully justify a belief in a proposition, and hence, this possibility of error "must weaken still further our first evidence." And, Hume wishes to claim, since "no finite object can subsist under a decrease repeated in infinitum," eventually there will remain nothing of the original probability.

Strictly speaking, Hume would want to claim that the probability of Q approaches a limit of zero as n goes to infinity, and hence we would have no justification (based on reason) for believing that proposition. So if reason were the basis of our beliefs, then we would have no beliefs.

However, the second reconstruction, as phrased, is also fallacious. It is fallacious because Hume's claim that "no finite object can subsist under a decrease repeated in infinitum" is false. Consider the number .99. Now decrease this number by .009. Take the new value and decrease it by .0009, then decrease this value by .00009, then by .000009, and so on. Contrary to Hume's claim, here is a situation where the original value does indeed subsist under a decrease repeated in infinitum. Not only does the original value not approach a limit of zero, it in fact never falls below .98.

A similar, though not so simple process can be described for the case of multiplication. Take our original value of .99, and multiply it by .999. Then take the new value, and multiply it by .99999, then multiply the resulting value by .9999999. By having each successive multiplicand be increasingly closer to one, it is possible to prevent the original value from approaching a limit of zero. In fact, by such means it is again possible to keep the value above .98. In short, vis-à-vis the second reconstruction above, we can see that while each consideration of new evidence E1, E2, E3, etc. does decrease the probability of Q, it does not follow that the probability of Q approaches a limit of zero. Hence this second reconstruction, as phrased, is also invalid.

However, it should be clear that in order to keep the probability of Q from approaching a limit of zero, it is necessary to assign probabilities which get increasingly closer to one. As such, we can make the second reconstruction valid by simply restricting the probabilities to some appropriate value less than one.[8] Here is where we return our attention to (P1), offering in its place

P1.1) Every proposition has a probability less than .999999999999999

Turning our attention back to Reconstruction 2, we can see that if (p1.1) is used in the reconstruction in place of (P1), then the probability of Q does indeed approach a limit of zero. In short, Reconstruction 2, using (P1.1), is a valid argument with no obviously false premisses.

Allow me to summarize before moving on. We have thus far established that a) Hume's probability argument, as presented in the Treatise I,iv,1, is almost certainly fallacious (assuming that Hume's argument was intended to be along the lines of Reconstruction 1 or Reconstruction 2), but b) the argument is not as bad as some commentators seem to think, since c) there is a way of reconstructing Hume's probability argument, using (P1.1) in place of (P1), such that the argument is valid and contains no obviously false premisses. I will next briefly discuss (P1) and (P1.1).

Of particular interest is whether it would be open to Hume to use (P1.1) in place of (P1). We must first remember that (P1), as phrased, does not appear in the argument of I,iv,1; rather, (P1) is intended to capture Hume's sentiments that no proposition is certain. The question, then, is whether (P1.1) captures these same sentiments.

It seems that a reasonable argument can be given to the effect that (P1.1) does indeed capture Hume's sentiments that no proposition is certain. Hume's sentiments here seem to involve a feeling of certainty;[9] his arguments at the beginning of I,iv,I seem designed to show that we should never feel completely certain of any proposition. I see no reason why Hume could not argue that we should never assign a probability to any proposition higher than .999999999999999, on the grounds that, as a matter of psychological fact, our feeling of certainty toward a proposition with a probability of .999999999999999 is no different than our feeling of certainty toward a proposition with a probability of one. So from the standpoint of 'feelings of certainty', it might be that (P1.1) captures Hume's sentiments as well as (P1).[10] As such, Reconstruction 2, using (P1.1) in place of (P1), is a valid argument, consistent with Hume's text, which contains no obviously false premisses.

One task remains before closing. Fred Wilson has recently offered a reconstruction of Hume's probability argument, and in so doing has argued to essentially the same conclusion as I; or as he puts it, Hume's probability argument is "worthy of a philosopher, and if it is unsound, then it is at least one that is worth refuting."[11]

In outline, Wilson's defense of Hume's argument is as follows: we know from probability calculus reasoning that in a chain of hearsay testimony, the probability of a proposition decreases as we get further removed from the original testimony. That is, suppose person 1 is the sole witness of John murdering

Sam, and passes this information on to person 2, who passes it on to person 3, who passes it on to person 4, and so on. At each link in this chain, the probability that the information is accurate decreases, given that the probability is based only on the testimony of that link (this is, of course, the reason for the ban on hearsay testimony in courtrooms). If this chain of testimony goes on indefinitely, then the probability of the proposition will tend to zero. Wilson maintains that Hume's probability argument of I,iv,1 is of the same sort. As the argument concerning the regress of probabilities involving hearsay testimony is valid, so Hume's argument is likewise valid.

There are two issues to be kept in mind here: a) did Hume intend his argument of I,iv,1 to be of the same sort as the hearsay argument?, and b) regardless of Hume's intentions, can the argument of I,iv,1 be reasonably reconstructed along the lines of the hearsay argument? The questions are intertwined and I will address them together, arguing that the answer to both should be 'no' and hence Wilson's reconstruction will not salvage Hume's probability argument.

It is worth noting that Hume clearly does present a version of the hearsay argument earlier in the Treatise, in the section entitled 'Of unphilosophical probability', (T 143-146) and furthermore, in this section he references the argument of I,iv,1 via a footnote.[12] Nonetheless, there are numerous reasons for doubting that Hume saw the arguments as being the same. First and foremost is the fact that in the hearsay argument, the probability of a proposition tends to zero only in the absence of the original testimony. In the "Of unphilosophical probability" argument Hume's concern is with ancient history which has been passed on through a chain of testimony over the years. Here all the ingredients of the hearsay argument are present; there is a long chain of testimony, the testimony being passed from one person to another, and in which the original testimony is absent.

But these critical ingredients are conspicuously absent in the argument of I,iv,1; in particular, no mention is made of testimony being passed on, and certainly there is no testimony passed from person to person. One could perhaps get something approaching a chain of testimony by letting memory play the role of the links in a chain of hearsay testimony.[13] But there are two substantial problems with interpreting Hume's argument along these lines: a) if Hume intended his argument to be based on memory in this fashion, one would expect him to at least mention memory in the course of the argument, which he does not,[14] and b) viewing memory as analogous to the links in a chain of hearsay testimony is not the most reasonable view of memory. On this view of memory, memory would not be simply a copy of an impression; rather, it would be a copy which is recopied indefinitely, such that each recopy is analogous to the passing on of testimony in the hearsay argument. But this is just an odd view of memory; what seems more reasonable is to view memory as being simply a copy of an impression (a copy which may fade, so to speak, but not one which is continually recopied).[15] And on this view of memory (which seems to be the view held by Hume)[16] memory is playing a role similar to that of "Printers and Copists". And printers and copists, as Hume points out, are capable of blocking the hearsay argument.[17]

In short, there seems to be too much disparity between the "Of unphilosophical probability" argument and the argument of I,iv,1 for Hume to have intended them to be the same argument. Furthermore, the argument of I,iv,1 cannot be reasonably reconstructed along the lines of the hearsay argument without adopting the non-Humean view of memory discussed above. And so it seems that Wilson's reconstruction will not salvage Hume's argument of I,iv,1.

At bottom, however, it is not terribly critical whether Hume intended his probability argument of I,iv,1 to be of the same form as the hearsay argument, or intended it to be along the lines of the reconstructions presented in this paper. For in either case the conclusion shared by Wilson and me is justified: there is a reconstruction of Hume's argument, consistent with Hume's text, which is valid and contains no obviously false premisses.

Whether Hume's argument is sound, and (as concerns my reconstructions) whether or not (Pl.1) successfully captures Hume's sentiments that no proposition is certain, are open questions which I do not wish to take up here. Again, my contention in this paper has been that there is a construal of Hume's probability argument of I,iv,1 which, if not sound, is at least valid and contains no obviously false premisses. Whether or not this argument is in fact sound is a question best left for another day.[18]

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1. [4], pp. 180-184.

2. [14], pp. 131-132.

3. Of the commentators listed below, only Popkin ([10], pp. 108-110) and Wilson ([19]) are clearly sympathetic toward Hume's argument, while Von Wright ([16], p. 153) seems to agree with the argument. Stove ([14], pp. 131-132) and Laird ([6], p. 176) clearly think the argument is defective, while Kemp-Smith ([13], pp. 443-450),

Penulhum ([9], pp. 253-278), Church ([2], pp. 182-184), Wilbanks ([18], pp. 134-141), Norton ([7], pp. 223-227), and Immerwahr ([5], pp. 227-230), discuss the argument but give no clear opinion as to whether or not they think the argument is defective. For a more thorough discussion of attitudes toward this argument, see Wilson ([19], pp. 101-105).

4. Wilson ([19]) has recently argued to essentially the same conclusion. His reconstruction of Hume's argument is discussed toward the end of the present paper.

5. Of course, (P1) is not obviously true, either. For some qualms concerning this claim, and Hume's use of it, see [6], p. 104. It should also be noted that, as phrased, (P1) does not appear in I,iv,1; rather, (P1) is simply a convenient way to capture Hume's sentiment that no proposition is certain.

6. 'Probability' is commonly used in two distinct ways: a) 'probability' as meaning 'likelihood of truth', and b) 'probability' as a measure of justification of belief. My point in this passage is that, on either reading of 'probability', the term 'evidence' usually refers to something which we believe to have a probability of one.

7. In fact, the textual evidence seems neutral between which of these reconstructions Hume intended (assuming he intended one or the other). His talk of possibility of error in our estimations seems more in line with the first reconstruction; while the several references to weakening of evidence seems more in line with the second reconstruction.

8. There is nothing special about the value chosen for (P1.1); any value less than one will do.

9. See [19], pp. 103-104, for further discussion of felt certainty.

10. This is not intended as an argument that (P1.1) does in fact capture these sentiments; rather, it is merely a suggestion that such an argument might be given.

11. [19], p. 124.

12. Hume's footnote, however, provides no support for viewing the 'Of unphilosophical probability' argument and the argument of I,iv,1 as being the same. His footnote simply points out that in

I,iv,1 we will find an exception to the general principle that "however great that conviction may be suppos'd, 'tis impossible it can subsist under such reiterated diminutions" ([4], p. 145. The exception, by the way, is that while reason dictates that the probability continually diminishes, we nonetheless retain our conviction or belief in the proposition. This shows, Hume thinks, that reason is not the basis of our convictions and beliefs). While obviously both sections are concerned with reiterated diminutions, there is no evidence that the diminutions are the result of the same reasoning. And as I am arguing in the present paper, there is good reason to think that the diminutions are not the result of the same reasoning.

13. This possibility was suggested by an anonymous referee.

14. The closest Hume comes to mentioning memory in the argument of I,iv,1 is when he points out that even an expert "must be conscious of many errors in the past, and must still dread the like for the future." ([4], p. 182). But here he is simply supporting his view that nothing is certain, only probable.

15. It should be pointed out that one cannot get a regress of probabilities tending to zero from simply the fact that memory is fallible (that is, from the fact that the copy fades, or loses its vivacity). Suppose I assign a probability of .99 to the proposition 'the sun will rise tomorrow,' based on evidence such as memory of past sunrises, my belief that the future will resemble the past, and so on. The fact that memory is fallible may cause me to question whether this is the correct assignment of probability, and I may decide that this assignment has only a .99 probability of being correct. Letting Q stand for the proposition in question, what we have is $P(P(Q) = .99) = .99$. That is, what we have going here is an argument of the form of Reconstruction 1 above, which as pointed out is clearly invalid. So the fallibility of memory alone will not get the needed regress of probabilities; rather, to get such a regress from an argument based on memory, memory must be playing a role analogous to the links in the chain of hearsay testimony.

16. "'Tis evident, that the memory preserves the original form, in which its objects were presented, and that where-ever we depart from it in recollecting any thing, it proceeds from some defect or imperfection in that faculty." ([4], p. 9).

17. It is printers and copists which block the hearsay argument as it is applied to ancient history (see [4], p. 146). If the argument of I,iv,1 were based on memory, and if memory played a role similar to printers and copists, then one would expect the argument of I,iv,1 to likewise be blocked.

18. The author wishes to thank Alan Haussman and George Schumm for helpful comments on earlier drafts of this paper.

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