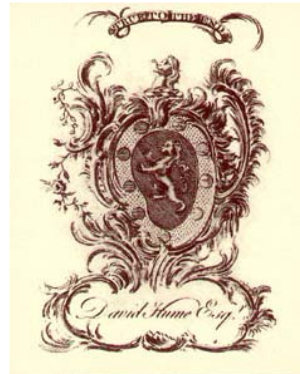

Kant's Misrepresentations of Hume's Philosophy of Mathematics in the Prolegomena

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KANT'S MISREPRESENTATIONS OF HUME'S PHILOSOPHY OF MATHEMATICS IN THE PROLEGOMENA

In 1783, Immanuel Kant published the following reflections upon the philosophy of mathematics of David Hume, words which have colored all subsequent interpretations of the latter's work:

> Hume being prompted to cast his eye over the whole field of a priori cognitions in which human understanding claims such mighty possessions (a calling he felt worthy of a philosopher) heedlessly severed from it a whole, and indeed its most valuable, province, namely, pure mathematics; for he imagined its nature or, so to speak, the state constitution of this empire depended on totally different principles, namely, on the law of contradiction alone; and although he did not divide judgments in this manner formally and universally as I have done here, what he said was equivalent to this: that mathematics contains only analytical, but metaphysics synthetical, a priori propositions. In this he was mistaken, and the mistake had a decidedly injurious effect upon his whole conception. But for this, he would have extended his question concerning the origin of our synthetical judgments far beyond the metaphysical concept of causality and included in it the possibility of mathematics a priori also, for this latter he must have assumed to be equally synthetical. And then he could not have based his metaphysical propositions on mere experience without subjecting the axioms of mathematics equally to experience, a thing which he was far too acute to do. The good company in which metaphysics would thus have been brought would have saved it from the danger of a contemptuous ill-treatment, or the thrust intended for it must have reached mathematics, which was and could not have been Hume's intention. Thus that acute

> man could have been led into considerations which must needs be similar to those that now occupy us, but would have gained inestimably by his inimitably elegant style.[1]

In other words, Hume failed to notice that mathematics, despite its _a priori_ character, is synthetic. It was thus easy for him to conclude that all _a priori_ judgments are analytic, and that therefore metaphysics (conceived of as synthetic _a priori_ judgments) is impossible.

Now what considerations led Kant himself to the opposite conclusion, that mathematics is synthetic despite being _a priori_? Let's consider Kant's own words in a typical passage:

> Just as little is any principle of geometry analytical. That a straight line is the shortest path between two points is a synthetical proposition. For my concept of straight contains nothing of quantity, but only a quality. The concept 'shortest' is therefore altogether additional and cannot be obtained by any analysis of the concept 'straight line.' Here, too, intuition must come to aid us. It alone makes the synthesis possible.

I should like the reader now to compare the following passage from Hume's _Treatise of Human Nature_, Book I, Part II, Section IV:[2]

> 'Tis true, mathematicians pretend they give an exact definition of a right line, when they say, _it is the shortest way betwixt two points_. But in the first place, I observe, that this is more properly the discovery of one of the properties of a right line, than a just definition of it. For I ask any one, if upon mention of a right line he thinks not immediately on such a particular appearance, and if 'tis not by accident only that he considers this property? A right line can be comprehended alone; but this definition is unintelligible

> without a comparison with other lines, which we conceive to be more extended. In common life 'tis establish'd as a maxim, that the streightest way is always the shortest; which wou'd be as absurd as to say, the shortest way is always the shortest, if our idea of a right line was not different from that of the shortest way betwixt two points.

Style and terminology apart, I defy the reader to distinguish between these two arguments and their conclusions. Hume argues that 'The shortest distance between two points is a straight line' is no definition; furthermore he says almost explicitly that 'There is a longer distance between two points than a straight line' is not a logical contradiction. And this amounts to saying that 'The shortest distance between two points is a straight line' is a synthetic judgment! One is driven to the conclusion that Kant was not aware of this statement of Hume's.

What, then, did lead Kant to his characterization of Hume's philosophy of mathematics. Why were subsequent generations taken in by Kant's mistake?

Well, there is the distinction Hume draws in the Enquiry Concerning Human Understanding, Section IV, Part I, paragraph 20:[3]

> All the objects of human reason or enquiry may naturally be divided into two kinds, to wit, Relations of Ideas, and Matters of Fact. Of the first kind are the sciences of Geometry, Algebra, and Arithmetic; and in short, every affirmation which is either intuitively or demonstratively certain. That the square of the hypothenuse is equal to the square of the two sides, is a proposition which expresses a relation between these figures. That three times five is equal to the half of thirty, expresses a relation between these numbers. Propositions of this kind are

> discoverable by the mere operation of thought, without dependence on what is anywhere existent in the universe. Though there never were a circle or triangle in nature, the truths demonstrated by Euclid would for ever retain their certainty and evidence.

Now does this passage lend any credence to Kant's assertion that Hume's view is that mathematical propositions are analytic? Not at all. All Hume is saying, if we adopt the Kantian terminology in discussing Hume, is that mathematical knowledge is _a priori_! In fact, this terminology is not foreign to Hume himself, as we gather from paragraph 23; for in speaking of the causal relation, Hume says:

> I shall venture to affirm, as a general proposition, which admits of no exception, that the knowledge of this relation is not, in any instance, attained by reasonings _a priori_; but arises entirely from experience, when we find that any particular objects are constantly conjoined with each other. Let an object be presented to a man of ever so strong natural reason and abilities; if that object be entirely new to him, he will not be able, by the most accurate examination of its sensible qualities, to discover any of its causes or effects. (E 27)

Hume here is implying that mathematical knowledge is _a priori_ in exactly the Kantian sense of 'not based upon experience.' We have already seen that to arrive at knowledge by comparing two ideas does not give us definitional (analytic) truth. It does, however, bypass experience.

It may be objected that Hume, in the passage before, equates the _a priori_ with what is certain, including geometry in the bargain. Yet Hume _denies_, in the _Treatise_, Part II, that geometry is certain. Since his doctrine in the _Enquiry_ contradicts his doctrine in the Treatise, we should ignore the latter, and attempt to interpret the Enquiry along Kantian lines.

I do not believe, however, that there is any contradiction between the philosophy of mathematics taught in the Treatise and that taught in the Enquiry. It is true that in Book I, Part II, Section IV Hume challenges the mathematician to disclose

> ...what infallible assurance he has, not only of the more intricate and obscure propositions of his science, but of the most vulgar and obvious principles? How can he prove to me, for instance, that two right lines cannot have one common segment? Or that 'tis impossible to draw more than one right line betwixt any two points? Shou'd he tell me, that these opinions are obviously absurd, and repugnant to our clear ideas; I wou'd answer, that I do not deny, where two right lines incline upon each other with a sensible angle, but 'tis absurd to imagine them to have a common segment. But supposing these two lines to approach at the rate of an inch in twenty leagues, I perceive no absurdity in asserting, that upon their contact they become one.... (T 51)

But Hume's argument here is simply to prove

> ...the fallacy of geometrical demonstrations, when carry'd beyond a certain degree of minuteness.... (T 53)

In other words, Hume's real doctrine is that the theorems of geometry are not strictly true -- a point of view which antedates that of Einstein. In writing a popular book like the Enquiry, which in any case was more about empirical than a priori knowledge, Hume skipped the refinements of the Treatise. Note, however, that Hume is still entitled to his view in the Enquiry that mathematics is a priori. For even on his view of the Treatise that geometry is not strictly true, the propositions of geometry could be rephrased in a way which would make them strictly true. One simply could say, as Einstein said: the sum of the angles of a triangle is closer and closer to 180 degrees, the smaller the triangle is. And *this* proposition is true, *and* -- since it is arrived at by comparing ideas -- *a priori*.

In sum, then, with respect to the characterization of mathematics as synthetic, *a priori* knowledge, there is no difference whatever between Kant and Hume. There is, presumably, a difference between them concerning the exact *truth* of geometrical statements. But if Hume is right concerning the phenomenology of space, it would seem that Kant has no argument against him -- as Charles Parsons pointed out long ago. For no Kantian argument can contradict the evidence of the senses by appealing to a 'higher tribunal.' Kantian arguments, presumably, can only alter the *modality* of known propositions -- they can explain why certain of our propositions are necessarily the way the senses tell us that they are.

If, then, there is any distinction between Kant and Hume concerning the philosophy of mathematics, it must lie here -- in the *modality* of mathematics. And, in fact, there is a big difference between the two in this regard. Kant regards mathematics as self-evidently *necessary*. In fact, this necessity is Kant's major argument for the *a priori* nature of mathematics.

This distinction is similar to the case of the concept of causality. Kant's major complaint against Hume's treatment is that it does not distinguish between subjective 'judgments of perception' and objective 'experience.' Hume does not distinguish between "When the sun shines on the stone, it grows warm" and "The sun warms the stone."[4] For Kant the difference lies in the necessity and universality of the latter:

> In order to test Hume's problematical concept (his crux metaphysicorum, the concept of cause), we are first given a priori, by means of logic, the form of a conditional judgment in general; that is, we have one cognition given as antecedent and another as consequent. But it is possible that in perception we may meet with a rule of relation which runs thus: that a certain appearance is constantly followed by another (though not conversely); and this is a case for me to use the hypothetical judgment and, for instance, to say if the sun shines long enough upon a body it grows warm. Here there is as yet no necessity of connection or concept of cause. But I proceed and say that, if this proposition, which is merely a subjective connection of perceptions, is to be a proposition of experience, it must be seen as necessary and universally valid. Such a proposition would be that the sun is by its light the cause of heat. The empirical rule is now considered as a law, and as valid, not merely of appearance but valid of them for the purposes of a possible experience which requires universal and therefore necessarily valid laws.[5]

It would not, perhaps, be too great a distortion to summarize Kant's view here in contemporary terminology by saying that the notion of a causal statement is derived from the concept of a hypothetical generalization of the form

For all x, if Fx then Gx

where the x ranges over all possible perceptions. Thus, Kant believes, Hume's 'problem' is solved: the concept of cause is derived from pure logic alone. (How we come to know particular causal judgments is another matter -- since this article is about Hume, we shall not pursue it.) Similarly for mathematics: for Kant, geometry holds _a priori_ for all _possible_ 'intuitions' -- Hume does not seem to hold that view.

What Kant failed to notice, however, is that Hume by no means could have adopted any view that invoked _possible_ perceptions, or, adopting now the Humean terminology, possible _impressions_. It is true that Hume's discourse is full of modal terms -- what is imaginable, for example, is what for him is possible.[6] But note that what is imaginable is what is conceived of through what Hume calls "complex ideas" -- for example (Hume's example) the "New Jerusalem" is possible because it can be constructed in the mind by rearranging various simple ideas. The (nonexistent) impressions corresponding to these ideas are, first of all, complex impressions; more importantly, they are impression-types, not impression-events.

What I deny is that Hume can have any notion of a possible, but not actual, simple impression-event. The concept of an impression itself, of course, is an idea of "reflexion" derived from observing our own mental powers. But a simple Humean thought experiment will convince us that "possible simple impression" is an empty phrase: conceive any simple impression-event;[7] then, conceive of the same impression as possible but not actual. The mental acts are phenomenologically identical. (Naturally, this is a spin-off from Hume's critique of the concept of existence.)

For Hume, then, modal distinctions collapse: it adds nothing to the proposition that the base angles of an isosceles triangle are equal, to say that this proposition is necessary. The proposition is _a priori_ -- derived from nonempirical comparison of various simple impressions. But on Hume's philosophy, it makes no sense to say that this comparison holds good, not only of actual impressions, but also of possible impressions. Similarly, it makes no sense for Hume to distinguish between true empirical generalizations and true generalizations that hold of necessity.

Hume could have used the concept of a possible impression. For, as H.H. Price already noted, Hume's concept of the external world is badly flawed without it. Beyond the arguments to this effect that Price gives in his wonderful book, I have noticed that Hume's philosophy is totally incoherent without such a concept. For Hume holds explicitly in the _Treatise_ (T 212) that the causal relation is between impressions and not objects (that is why he cannot accept the Cartesian causal theory of perception). Further, he holds that causation is nothing but constant conjunction -- hence, constant conjunction of impressions. But, third, he holds that we can reason from effect to cause, as well as from cause to effect. (T 173ff) That is, we can infer the existence of causes that we have had no direct experience of, as when we infer the existence of extinct species from their remains -- in the same manner as we reason from present causes to effects that we have not yet seen. All these propositions add up to a blatant contradiction -- since for Hume, all causes have to be impressions. And in the case of mammoths and dinosaurs the whole point is that we have _not_ had impressions of them. Hence, strictly speaking, fossils have no cause.

There are two ways to escape this trap. One is to accept the existence of 'unsensed sensibilia,' impressions actually existing but which no one has perceived. As Price points out, this is not as ridiculous -- for Hume -- as it might seem, as for Hume the entire mind is constituted as a "bundle of perceptions." Unsensed sensibilia can easily be grasped by imagining the bundle disappearing except for one perception. Nevertheless, as Price also points out, Hume explicitly rejected this possibility on empirical grounds -- this is the import of the "double vision" experiments in Part IV. (T 210-211)

This leaves the concept of a possible but not actual impression as the only way out of the incoherence we have detected in Hume's doctrine of the external world. That is, a cause is an actual or possible impression which stands in the appropriate relation of constant conjunction to the effect -- which is also an actual or possible impression. (Remember the doctoral dissertation about the influence of the unpublished dialogues of Plato on the last works of Aristotle?) This would certainly solve the problem -- but, as I have already argued, this was a route not open to Hume. It is for this reason that Hume, in discussing the unseen door, "turning upon its hinges," (T 196) systematically refuses to invoke possible door-perceptions in explaining our inclination to believe in the continuing existence of unperceived objects. According to our view, Hume was entirely correct not to pursue this line of reasoning, as the notion of a "possible but not actual" impression-event was not available to him.

The true Humean philosophy of mathematics, then, was different from that of Kant, but in ways that Kant himself did not imagine. Both Kant and Hume held mathematics to be synthetic and a priori; where they differed, was in the nature of mathematical necessity, and its relation to the mathematical a priori. For Hume, mathematical truth and mathematical necessity were one and the same -- modal distinctions collapse. For Kant, it was philosophically crucial that mathematical theorems were necessary in a sense which transcend the merely true.

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1. Immanuel Kant, Prolegomena to Any Future Metaphysics, trans. Lewis White Beck (Indianapolis and New York: The Library of Liberal Arts, 1950), pp. 17-18.

2. David Hume, A Treatise of Human Nature, ed. P.H. Nidditch (Oxford: Oxford University Press, 1978), pp. 49-50. Further references will be cited as 'T' followed by the relevant page number(s).

3. David Hume, Enquiries Concerning Human Understanding and Concerning the Principles of Morals, ed. L.A. Selby-Bigge, 3rd ed. with text revised and notes by P.H. Nidditch (Oxford: Clarendon Press, 1975), p. 25. Further references will be cited as 'E' followed by the relevant page number(s).

4. Prolegomena, p. 49.

5. Prolegomena, p. 59.

6. I realize that there are problems with this formulation of Hume's, since a term with a double modality is equated to one with a single modality; but I cannot stop to discuss this here in any length.

7. From now on, when I write "impression" I shall mean "impression-event"; i.e., the "token," to use the jargon.