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DALE JACQUETTE

The Limitations of Reason

The arguments against infinite divisibility in the notes to Sections 124 and 125 of David Hume's *Enquiry Concerning Human Understanding* are presented as "sceptical" results about the limitations of reason. The metaphysics of infinite divisibility is introduced merely as a particular, though especially representative problem, among several that Hume discusses. Hume first writes:

The chief objection against all *abstract* reasonings is derived from the ideas of space and time; ideas, which, in common life and to a careless view, are very clear and intelligible, but when they pass through the scrutiny of the profound sciences (and they are the chief object of these sciences) afford principles, which seem full of absurdity and contradiction.¹

There follows an intuitive appeal to the apparent incoherence of the consequences of infinite divisibility in the geometry of space and measurement of time. Hume enlists natural belief against the infinite divisibility thesis in the mathematics of extension:

But what renders the matter more extraordinary, is, that these seemingly absurd opinions are supported by a chain of reasoning, the

clearest and most natural; nor is it possible for us to allow the premises without admitting the consequences. Nothing can be more convincing and satisfactory than all the conclusions concerning the properties of circles and triangles; and yet, when these are once received, how can we deny, that the angle of contact between a circle and its tangent is infinitely less than any rectilinear angle, that as you may increase the diameter of the circle *in infinitum*, this angle of contact becomes still less, even *in infinitum*, and that the angle of contact between other curves and their tangents may be infinitely less than those between any circle and its tangent, and so on, *in infinitum*? The demonstration of these principles seems as unexceptionable as that which proves the three angles of a triangle to be equal to two right ones, though the latter opinion be natural and easy, and the former big with contradiction and absurdity. (EHU 156-157)

The implied sense of contradiction is then extended by Hume to the problem of infinite divisibility in time.

The absurdity of these bold determinations of the abstract sciences seems to become, if possible, still more palpable with regard to time than extension. An infinite number of real parts of time, passing in succession, and exhausted one after another, appears so evident a contradiction, that no man, one should think, whose judgment is not corrupted, instead of being improved, by the sciences, would ever be able to admit of it. (EHU 157)

The contradictions involved in traditional concepts of extension and time demarcate the limits of which reason is capable. But Hume's answer is more complex than merely surrendering to skepticism over the prospects of reason achieving understanding in the metaphysics of space and time. He tries to show that what he calls "Pyrrhonian" skepticism is ultimately self-defeating in its application of reason against reason. In place of a negative conclusion, he offers another solution to the paradoxes of infinite divisibility.

What is remarkable about Hume's pronouncements against the infinite divisibility of extension and time, that may signal a change in his methodology if not his overall position on divisibility in the *Enquiry* from that of *A Treatise of Human Nature*, is his straightforward claim that natural belief rebels against the concept of infinite divisibility as its consequences apply to the metric of space and time.² As in the *Treatise*, Hume pits natural disbelief in infinite divisibility against abstract metaphysical reason. But in the *Enquiry*, he does not invoke reason merely to correct the excesses of natural belief for the skeptical deflation of natural belief, say, in the necessity of causal connection, or the existence of mind-independent continuants. On the contrary,

Hume here appears instead to invoke natural disbelief in infinite divisibility as a weapon against what he regards as the immoderate and unjustified deliberations of speculative reason in traditional mathematics and metaphysics.

Hume holds that natural belief and metaphysical reason may coincide. This occurs when reason is exercised in philosophical inquiry and the examination of arguments for and against a position produces a true metaphysics tempered by instinct and intuition. In the *Treatise*, with respect to the natural beliefs about which Hume prefers to remain skeptical, the appeal to pretheoretical natural belief is no deterrent to or substitute for painstaking metaphysical investigations. But in the first *Enquiry*, Hume seems content to set aside doubts about natural beliefs that run counter to the infinite divisibility of extension and time where his own previous reasonings in the *Treatise* happen to agree. That this is actually an incorrect assessment of Hume's procedure in the *Enquiry* appears only when it is observed that the conflict between natural belief and philosophical reason is a two-edged sword that cuts equally against natural belief and metaphysical reason to reinforce the skeptical attitude whenever they collide. Hume's purpose in these brief notes in the *Enquiry* is not simply to offer a naive intuitive bulwark against infinite divisibility that uncritically takes natural disbelief in infinite divisibility as sufficient leverage to overturn abstract metaphysical commitment to infinity. This would violate Hume's practice in the *Treatise* of using metaphysics to correct the extravagances of natural belief. Hume's objective is rather to illustrate the general thesis that skepticism arises whenever natural belief conflicts with metaphysical reason.

This important difference in emphasis can be seen in the fact that Hume's arguments against infinite divisibility are presented in asides to the main body of the text. By contrast with the *Treatise* Book I, Part 2, this format indicates that Hume's aim in the chapter is not primarily to refute the infinite divisibility thesis, but instead to address questions posed at the beginning of the *Enquiry*, Section XII, "Of the academical or sceptical Philosophy." These are the problems, in Hume's words, of "What is meant by a sceptic? And how far is it possible to push these philosophical principles of doubt and uncertainty?" (EHU 149). The *Treatise*, by comparison, does not introduce the topic of skepticism at length until 112 pages after the discussion of infinite divisibility, in Book I, Part 4, "Of the sceptical and other systems of philosophy."

Self-Defeating "Pyrrhonian" Skepticism

Hume distinguishes several kinds and categories of skepticism. He assesses the philosophical strengths and limitations of each, and arrives at last at the statement to which the note containing his second argument against infinite divisibility in *Enquiry* Section 125 is attached. Here he declares:

Yet still reason must remain restless, and unquiet, even with regard to that scepticism, to which she is driven by these seeming absurdities and contradictions. How any clear, distinct idea can contain circumstances, contradictory to itself, or to any other clear, distinct idea, is absolutely incomprehensible, as absurd as any proposition, which can be formed. So that nothing can be more sceptical, or more full of doubt and hesitation, than this scepticism itself, which arises from some of the paradoxical conclusions of geometry or the science of quantity. (EHU 157–158)

Hume first observes the conflict between natural disbelief in infinite divisibility and the conclusions that follow unavoidably from the apparently innocent premises that serve as starting-places and logically circumspect chains of inference in abstract reasoning in mathematics and metaphysics. This does not by itself constitute an argument against infinite divisibility, nor does Hume offer it as such. His point is that skepticism is self-defeating when it tries to use reason to undermine reason. Hume begins Part II of the Section with the statement that: "It may seem a very extravagant attempt of the sceptics to destroy reason by argument and ratiocination; yet is this the grand scope of all their enquiries and disputes" (EHU 155). Sceptics try to do this, he continues, when "[t]hey endeavour to find objections, both to our abstract reasonings, and to those which regard matter of fact and existence" (EHU 156).

Then Hume introduces the criticism reproduced above, that "The chief objection against all abstract reasonings is derived from the ideas of space and time...." His diagnosis is that a paradox occurs when skeptics try to confute abstract reasoning by a *reductio ad absurdum*, using well-reasoned arguments that derive from what may appear to be irreproachable premises and logically valid inference chains the self-contradictory conclusions that extension and time are infinitely divisible. The rationale for regarding the conclusion as absurd or self-contradictory is not given, beyond what amounts to Hume's assertion that these results are inconsistent with natural belief about the divisibility of space and time, and the *reductio* argument discussed below in the note to Section 124. The skeptic, in observing the absurdity of the consequences of abstract reason in the metaphysics of infinitely divisible space and time, indiscriminately casts doubt on reason in general, on the grounds that the conflict between natural and philosophical reason indicates that neither form is entirely to be trusted.

The note to *Enquiry* 124 contains Hume's first criticism of infinite divisibility. It is a *reductio* offered in the guise primarily of an example of how skepticism seeks to undermine abstract reasoning. Hume is unequivocal in the argument he gives to support the skeptic's claim in the passage to which the note is attached. He maintains:

A real quantity, infinitely less than any finite quantity, containing quantities infinitely less than itself, and so on *in infinitum*; this is an edifice so bold and prodigious, that it is too weighty for any pretended demonstration to support, because it shocks the clearest and most natural principles of human reason. (EHU 156)

The skeptic's objection to abstract reasoning is interpreted by Hume significantly as the complaint that it implies a contradiction with "human reason." This may seem to elevate ordinary thought to the position of a standard by which the intelligibility of abstract metaphysical reasoning can be criticized. But Hume explains the goal of skeptics more generally as the attempt to "destroy reason by argument and ratiocination," or to destroy reason by means of reason. Hume regards global skepticism about these metaphysical matters as self-defeating and self-refuting. The view is further suggested in his previously noted pronouncement that "nothing can be more sceptical, or more full of doubt and hesitation, than this scepticism itself, which arises from some of the paradoxical conclusions of geometry or the science of quantity."

That skepticism in Hume's view is hoist by its own petard in trying to refute abstract reason as part of a misconceived or "extravagant" program to refute reason in general, appealing unavoidably to reason, "human reason" or "argument and ratiocination," is further indicated by the proofs in Hume's two notes about infinite divisibility. Here he by no means accepts (what is misleadingly termed) the "Pyrrhonian" skeptic's conclusion that human reason cannot rise above skeptical dilemmas about divisibility, but takes the opportunity instead to deliver two reasonable and sincerely intended arguments against infinite divisibility.³

This at once rescues reason from the Pyrrhonian skeptic's clutches. Hume's argument does not refute reason in general, a project he rightly regards as absurd, but attacks in particular the reasoning in traditional infinitist mathematics and metaphysics of space and time. Hume thereby reaffirms the empiricist rejection of infinite divisibility and theory of sensible extensionless indivisibles in the *Treatise*. Yet the argument should not be understood simply as trying to show that the infinite divisibility thesis contradicts natural belief as the final arbiter of metaphysical truth. That kind of argument would no more satisfy Hume in the *Enquiry* than in the *Treatise*, where philosophical reason is frequently called upon to correct the errors of natural belief. The appeal to natural reason or belief in the form of that "human reason" which contradicts the conclusions of abstract reasonings about space and time is left by Hume to the folly of the "Pyrrhonian" skeptic, entangled in the paradox of trying to refute reason by its own means.⁴

The *Reductio* of *Enquiry* 124

The disproof of infinite divisibility in the note to *Enquiry* 124 upholds reason against Pyrrhonian skepticism. It asserts that "nothing appears more certain to reason" than that infinitely many physical points must constitute an infinitely extended thing. Hume formulates the argument in this way:

Whatever disputes there may be about mathematical points, we must allow that there are physical points; that is, parts of extension, which cannot be divided or lessened, either by the eye or imagination. These images, then, which are present to the fancy or senses, are absolutely indivisible, and consequently must be allowed by mathematicians to be infinitely less than any real part of extension; and yet nothing appears more certain to reason, than that an infinite number of them composes an infinite extension. How much more an infinite number of those infinitely small parts of extension, which are still supposed infinitely divisible. (EHU 156, note 1)

The physical points beyond which neither "eye nor imagination" can discriminate recall the sensible extensionless indivisibles revealed by the *Treatise* inkspot experiment. This concept in the *Enquiry* 124 argument is grafted onto a *reductio* refutation of infinite divisibility appearing in the *Treatise* (and originally suggested by Bayle's *Dictionary* article on "Zeno of Elea").⁵ It will therefore be worthwhile to outline Hume's earlier discussion of both of these parts of the *Enquiry* 124 objection, before attempting a more complete critical exposition. Bayle had argued that the mind is incapable of understanding the divisibility of extension. His trilemma claims that there are only three imaginable ways to divide extension—infinately, into extended physical points, or into abstract extensionless mathematical points—each of which on examination is incoherent.

The first *Enquiry* argument against infinite divisibility begins with an appeal to Hume's concept of sensible extensionless indivisibles. The existence of phenomenal atoms or *minima sensibilia* individually lacking but collectively constituting extension was demonstrated in the *Treatise* inkspot experiment. Hume describes and interprets the procedure as follows:

Put a spot of ink upon paper, fix your eye upon that spot, and retire to such a distance, that at last you lose sight of it; 'tis plain, that the moment before it vanish'd the image or impression was perfectly indivisible. 'Tis not for want of rays of light striking on our eyes, that the minute parts of distant bodies convey not any sensible impression; but because they are remov'd beyond that distance, at which their impressions were reduc'd to a minimum, and were incapable of any farther diminution. (T 27)

The assumption in the argument of the first *Enquiry* 124 that extension is infinitely divisible, which Hume makes for purposes of indirect proof, is falsified by the contradiction that a finite extension is also infinitely extended. This conclusion is produced as in the *Treatise reductio* by considering the consequences of adding the infinitely many indivisible physical points of a finitely extended body together to produce an infinitely extended body. The idea is that if a finitely extended body is infinitely divisible, then it must be possible infinitely to add its parts one to another finally to constitute an infinitely extended body. Thus, we derive from the assumption of infinite divisibility the contradiction that every finitely extended body is infinitely extended. The problem is only made worse on the infinitist's premise that the infinite constituents of extension are infinitely divisible rather than indivisible. In the *Treatise*, Hume argues:

Every thing capable of being infinitely divided contains an infinite number of parts; otherwise the division would be stopt short by the indivisible parts, which we should immediately arrive at...But that this latter supposition is absurd I easily convince myself by the consideration of my clear ideas. I first take the least idea I can form of a part of extension, and being certain that there is nothing more minute than this idea, I conclude, that whatever I discover by its means must be a real quality of extension. I then repeat this idea once, twice, thrice, &c. and find the compound idea of extension, arising from its repetition, always to augment, and become double, triple, quadruple, &c. till at last it swells up to a considerable bulk, greater or smaller, in proportion as I repeat more or less the same idea. When I stop in the addition of parts, the idea of extension ceases to augment; and were I to carry on the addition in infinitum, I clearly perceive, that the idea of extension must also become infinite. Upon the whole, I conclude, that the idea of an infinite number of parts is individually the same idea with that of an infinite extension; that no finite extension is capable of containing an infinite number of parts; and consequently that no finite extension is infinitely divisible. (T 29-30)

The argument can be reconstructed⁶ in this way:

1. Suppose for purposes of indirect proof that some finitely extended thing is infinitely divisible into infinitely many (indivisible or extended) parts.
2. If a finitely extended thing is infinitely divisible into infinitely many (indivisible or extended) parts, then these parts could be added one to another *ad infinitum* to constitute a body of infinite extension, contrary to the assumption. (Also, each finitely extended thing would fill all of infinite space.)

3. Therefore, no finitely extended thing is infinitely divisible into infinitely many (indivisible or extended) parts. (1,2)
4. Therefore, space (extension) is not infinitely divisible. (3)

In the *Enquiry*, Hume similarly speaks of spatial extension as reducible to sensible extensionless indivisible "physical points," which he distinguishes from ideal or abstract Euclidean "mathematical points."⁷ The argument against infinite divisibility in *Enquiry* 124 is analogous to the *Treatise reductio* from the addition of infinite parts. Both derive the contradiction that a finitely extended body is infinitely extended, from the assumption that space is infinitely divisible. But in the *Enquiry*, Hume dispenses with the inkspot experiment as justification for the existence of sensible extensionless indivisibles, and begins with an equivalent assumption he believes even infinitist mathematicians are compelled to accept, that the physical points of finite extension are "infinitely less than any real part of extension."

The inference has the following indirect proof structure. It begins, like the *Treatise reductio*, with premises Hume does not accept, and others about which he remains neutral for the sake of argument, such as the existence of mathematical points.

1. There are physical points, sensible extensionless indivisible parts of extension.
2. Physical points are absolutely indivisible, in that if divided (separated) they would cease to be present to sensation or imagination.
3. Suppose for purposes of indirect proof that extension is infinitely divisible.

4. Physical points are infinitely less or infinitely smaller in extent than any real (extended) part of extension. (1,2)
5. Finitely extended things are divisible into infinitely many physical points. (3,4)
6. Infinitely many physical points are needed to constitute a finitely extended thing. (5)
7. Finitely extended things are also infinitely extended. (The problem is only compounded if extension is infinitely divisible into infinitely smaller infinitely divisible parts.) (2,6)
8. Therefore, space (extension) is not infinitely divisible. (3,7)

The proof requires a bit more reconstruction than most of Hume's *reductio* arguments in the *Treatise*. It appears appropriate to refer to "physical" points in Hume's argument as the sensible extensionless indivisible parts of extension, because he describes them in the text as "these *images*," which seems to imply their perceivability in sense experience or imagination (and also their ideality).

The assumption requires phenomenal evidence for the experienceable extensionless indivisibles established by the inkspot experiment. This may indicate alternatively that Hume here presupposes the empirical conclusions of the experiment, or that he now finds the proposition so obvious as not to require justification. The parenthetical remark in step (7) reverts and may also be intended as an allusion to the *reductio* argument from the addition of infinite parts in the *Treatise*. The proof is refined somewhat to leave open the question whether there are infinitely divisible mathematical points, and derives contradiction even from the more moderate assumption that there are sensible extensionless physical points, thereby concentrating attention more pointedly in particular on the offending assumption that extension is infinitely divisible.

The conclusions in (4) and (5) require additional explanation. That physical points are infinitely less or infinitely smaller in extent than any extended part of extension follows directly from assumptions (1) and (2). If physical points were not infinitely smaller than any extended part of extension, they would be at least the same size as the smallest extended parts of extension. Every extended thing is divisible, so that contrary to the definition in assumption (2), physical points would not be extensionless or indivisible unless they were infinitely smaller than the smallest extended parts of extension. This brings us to conclusion (5), that finitely extended things are divisible into infinitely many physical points.

The similarity between this condensed indirect proof against infinite divisibility and Hume's first *Treatise reductio* from the addition of infinite parts should not overshadow the fact that the *Enquiry reductio* represents a substantial improvement over its predecessor. It has already been observed that the latter demonstration claims an advantage in its neutrality about the existence of "mathematical" points. This allows the argument to discover the absurdity of infinite divisibility into indivisible physical points, which must be thought infinitely smaller than any least real parts of extension. The absurdity can then be extended with proportionately greater force to the traditional infinitist assumption that subsegments composed of infinitely many mathematical points, infinitely minute parts of extension, are themselves still infinitely divisible.

The *Enquiry* argument also has a distinct advantage over the *Treatise* proof in its common sense appeal to the existence of "physical" points, which are evidently the sensible experienceable extensionless indivisibles of the earlier theory. The *Treatise* finds it necessary to introduce indivisibles by the complicated apparatus of the inkspot experiment, with its contentious strong empiricist assumption about the experiential origin of adequate ideas in impressions of sensation. Here Hume says simply "we must allow that there are sensible extensionless physical points," a proposition he seems to believe even his infinitist adversary will accept. The point is not that the *Enquiry* 124

argument is superior to the *Treatise reductio* from the addition of infinite parts because it is always better simply to assume what it may be difficult or controversial to prove. Rather, Hume in the *Enquiry* appears to have found an argument that he regards as more economical but every bit as effective as the *Treatise* inkspot experiment. The same result is achieved without proving the existence of sensible extensionless indivisibles in the context of advancing a version of the *reductio* from the addition of infinite parts. Hume in the *Enquiry* notices that whether or not the infinitist also accepts the existence of infinitely divisible mathematical points, the argument from addition of infinite parts goes through on the basis of the infinitist's presumed acceptance of the existence of physical points that cannot be diminished by vision or imagination. For that matter, the inkspot experiment in the *Treatise* might also be construed not so much as a rigorous proof of the existence of sensible extensionless indivisibles, but as a heuristic visual aid to help explain and call attention in ordinary experience to what Hume means by the concept. If seriously challenged on the existence of physical points, Hume in the later argument might still want to invoke something like the data of the inkspot experiment to defend the assumption from objections, instead of relying entirely on prephilosophical opinion. But the fact that he does not do so explicitly is significant, since it indicates that he regards the argument as complete without further elaboration, and as a proof, moreover, that ought to convince even the traditional infinitist, without denying from the outset the possibility or conceivability that extension is composed of subsegments infinitely divisible into infinitely many mathematical points.⁸

To close the jaws of the *reductio* on "the mathematician" it might be expected that Hume would resort at this stage of the argument to the claim that physical points discriminated by finite vision or imagination in finite time cannot be infinite in number. This would bring the argument closer in spirit to the inkspot experiment argument of the *Treatise*. But he leaves the matter open, no doubt in keeping with his argumentative pose of neutrality on "whatever disputes there may be about mathematical points." Instead, Hume frames another, different, absurdity, to conclude against the infinitist that the assumptions lead inexorably to the contradiction that a finitely extended body composed of infinitely many physical points must be infinitely extended, on the grounds that "nothing appears more certain to reason, than that an infinite number of [physical points] composes an infinite extension." The problem is only made worse, though it is already plainly bad enough, if, as the mathematicians Hume criticizes are supposed to believe, extension is constituted by mathematical rather than physical points, infinitely small parts of extension, each of which in turn is infinitely divisible. The result in either case reflects back negatively on the hypothesis that extension is infinitely divisible.

Interestingly, Hume maintains that physical points are "absolutely

indivisible." This appears at first to contradict the divisibility or indivisibility of sensible ideas relative to subject and circumstance, the distance from subject to what is judged a physical point, the acuity of the eye, and the patience, memory, and focus of individual imagination. But perhaps by holding that the physical points are absolutely indivisible, Hume does not mean that phenomenal divisibility is absolute, but only that if divided-up or separated, physical points are indivisible, as he claims, to the "fancy or senses," in that they cease to be present to sensation or imagination. That, in any case, is all the argument seems to require.

Hume's Berkeleyan Rejection of Abstract Ideas in *Enquiry* 125

The same conclusion is reached in Hume's second *Enquiry* argument against infinite divisibility in the note to Section 125. This is not a *reductio*-style proof, but appears to offer a supplementary if not substitute inference for Hume's *Treatise* argument based on the inkspot experiment. The argument is also dependent in a less direct way on sensation and the empirical data of phenomenal experience, but derives directly from Berkeley's refutation of abstract general ideas. Hume offers the deduction in what looks curiously to be uncharacteristically tentative, hypothetical language.

It seems to me not impossible to avoid these absurdities and contradictions, if it be admitted, that there is no such thing as abstract or general ideas, properly speaking; but that all general ideas are, in reality, particular ones, attached to a general term, which recalls, upon occasion, other particular ones, that resemble, in certain circumstances, the idea, present to the mind. (EHU 158, note 1)

Hume advances the argument not primarily to dispute the infinite divisibility thesis, but chooses it almost randomly as an example to challenge the Pyrrhonian skeptic's global skepticism toward reason. The infinite divisibility thesis provides an opportunity for the skeptic to raise difficulties about abstract reasoning in the metaphysics of space and time. Hume in a single master stroke disables both the Pyrrhonian skeptic and the infinite divisibility of extension thesis. The proof contains Hume's famous "hint" against infinite divisibility, and, implicitly, in support of his doctrine of sensible extensionless indivisibles. Hume states:

...all the ideas of quantity, upon which mathematicians reason, are nothing but particular...and consequently, cannot be infinitely divisible. It is sufficient to have dropped this hint at present, without prosecuting it any farther. (EHU 158, note 1)

Although the argument does not appeal to anything like the inkspot experiment for empirical data about the limitations of divisibility or the experienceability of extensionless indivisibles, it rests ultimately on a kind of thought experiment against the existence of abstract ideas which Hume attributes to Berkeley.

Previously, in *Enquiry* Section XII, Part I, Hume portrays Berkeley as a skeptic about Locke's distinction between primary and secondary qualities. Hume agrees that this distinction can only be defended against skeptical objections if there exist abstract ideas. "Nothing can save us from this conclusion," he holds, "[that primary like secondary qualities are 'perceptions of the mind, without any external archetype or model, which they represent'], but the asserting, that the ideas of those primary qualities are attained by *Abstraction*, an opinion, which, if we examine it accurately, we shall find to be unintelligible, and even absurd" (EHU 154).

Hume challenges the reader, as Berkeley's Philonous challenges Hylas in *Three Dialogues Between Hylas and Philonous*, to conceive of an abstract general idea, of "triangle" or a triangle in general or in the abstract, of no particular size or type, with lines of no particular color.⁹ Like Berkeley, Hume concludes that the exercise must fail, and draws from this the moral that abstract ideas are philosophical fictions.

An extension, that is neither tangible nor visible, cannot possibly be conceived: and a tangible or visible extension, which is neither hard nor soft, black nor white, is equally beyond the reach of human conception. Let any man try to conceive a triangle in general, which is neither *Isosceles* nor *Scalenum*, nor has any particular length or proportion of sides; and he will soon perceive the absurdity of all the scholastic notions with regard to abstraction and general ideas. (EHU 154-155)

As in the *Treatise*, where he enthusiastically describes Berkeley's refutation of abstract ideas as "...one of the greatest and most valuable discoveries that has been made of late in the republic of letters..." (T 17), Hume in the first *Enquiry* accepts Berkeley's phenomenological thought experiment as showing that there can be no abstract general ideas. He adopts Berkeley's suggestion that what is called abstract reasoning involves reasoning from particular ideas delegated by the imagination to represent all members of the same relevant category, from which counterinstances will naturally come to mind if reasons in drawing conclusions for the entire set from properties peculiar to any of its specific representatives.¹⁰

This makes it strange to find Hume ranking Berkeley's discovery with the conclusions of skeptics in this part of the *Enquiry*. The ambiguities are evident in the note to Section 122:

This argument is drawn from Dr. Berkeley; and indeed most of the writings of that very ingenious author form the best lessons of scepticism, which are to be found either among the ancient or modern philosophers, Bayle not excepted. He professes, however, in his title-page (and undoubtedly with great truth) to have composed his book against the sceptics as well as against the atheists and free-thinkers. But that all his arguments, though otherwise intended, are, in reality, merely sceptical, appears from this, *that they admit of no answer and produce no conviction*. Their only effect is to cause that momentary amazement and irresolution and confusion, which is the result of scepticism. (EHU 155, note 1)

What is surprising is not so much that Hume should classify Berkeley as a kind of skeptic, but that in this particular context he claims that the arguments of skeptics like Berkeley should *produce no conviction*. This is strange because Hume himself appears convinced from the time of the *Treatise* that Berkeley's arguments against the existence of abstract general ideas are important and correct.¹¹

The passage admits of no easy interpretation. Hume undoubtedly wields Berkeley's pat refutation of abstract general ideas in the second argument against infinite divisibility in the *Enquiry*, which raises problems about the skeptical status of the argument. Yet the "hint" in Hume's second argument in the note to *Enquiry* 125 trades essentially on Berkeley's renunciation of abstract general ideas, together with the assertion that if mathematicians have no abstract ideas of quantity, but only particular ideas acquired from sensation and imagination, then no ideas are infinitely divisible.

...when the term Horse is pronounced, we immediately figure to ourselves the idea of a black or a white animal, of a particular size and figure: But as that term is also usually applied to animals of other colours, figures and sizes, these ideas, though not actually present to the imagination, are easily recalled; and our reasoning and conclusion proceed in the same way, as if they were actually present. If this be admitted (as seems reasonable) it follows that all the ideas of quantity, upon which mathematicians reason, are nothing but particular, and such as are suggested by the senses and imagination, and consequently, cannot be infinitely indivisible. (EHU 158)

The argument makes reference to the two main features of Berkeley's criticism, that there are no abstract general ideas, and that what is called abstract reasoning involves reasoning from particular ideas that represent other members belonging to the same category. The "hint" in the argument which Hume does not sufficiently develop, is formulated as the assumption

that no particular idea of extension and no idea of any particular extended thing is infinitely divisible. The proof can be reconstructed in this way.

1. There are no abstract ideas.
2. What is called abstract reasoning involves reasoning from particular ideas delegated by the imagination to represent the members of the same relevant category.
3. No particular idea of extension or idea of any particular extended thing is infinitely divisible.
4. No idea of extension is infinitely divisible; there is no adequate idea of extension in space as infinitely divisible. (1,2,3)

The "hint" of the argument is given here as assumption (3). It states that no particular idea of extension or idea of any particular extended thing is infinitely divisible. To pursue the hint requires conjecture about the kind of defense Hume might have offered in support of the proposition.

The assumption is crucial to the argument, though it is precisely the point Hume leaves to surmise. Yet it is plausible to suppose that Hume's purpose in the argument is satisfied by the fact that a finite mind making use of finite imagination in a finite amount of (finitely or infinitely divisible) real time cannot infinitely divide any particular idea of extension or particular idea of any extended thing.

The question is not whether the mind could *de re* conceive the idea of an infinitely divisible extension or infinitely divisible extended thing, but whether the idea itself could be infinitely divisible. But surely dividing a particular idea into infinitely many parts is beyond the limitations of the finite mind's memory, imagination, and reason, functioning in finite time. The feat could not be accomplished unless infinite time were available for the mind to attempt the subdivision with infinite imagination, memory, attention, and patience. But these assumptions cannot be introduced without begging the question against Hume's criticism of the very intelligibility of the concept of infinity.

The finite mind is certainly unable successively to subdivide a single idea of a particular extension or extended thing for more than a few passes, terminating in the substitution of another particular idea of extension or of another particular extended thing. This might then be further subdivided at greater imagined magnification. Or the process might end in frustration in the failure mentally to keep track of how many subdivisions have been performed, whether each part has been divided or overlooked, after, at best, say, the sixth or seventh operation. At this point, the mind's image is likely to degenerate from a clear distinct idea of an extended thing to a hazy outline of finite extension subdivided by indefinitely many cross-hatch striations.

This defense of Hume's assumption (3) agrees with the phenomenal

background of the proof in Berkeley's empiricist criticism of abstract general ideas, and the two conceptions seem indeed to go hand in hand. The presumption that the finite mind cannot accomplish the ideational task of infinitely dividing a particular idea of extension or idea of a particular extended thing powers the main assumption in Hume's argument against infinite divisibility as reconstructed. It complements the Berkeleyan skeptical conclusion that there are no abstract ideas, and hence no adequate abstract ideas of extension.

Another way to develop Hume's clue to the Berkeleyan refutation of infinite divisibility is to consider that if there are no abstract ideas, but only particular ones deputized to represent the category, then the particular idea of extension, or idea of a particular extended thing, must be something like a mental image of a finite length of a line, surface, or some material entity, of some finite imagined width or thickness. To have adequate ideas of such a particular finite length infinitely divided is impossible on what seem to be the only ways of imagining the division of a finite extension into component parts. The best method would be to mark the extension with imaginary short perpendicular line segments, cross-hairs, so to speak, subdividing the extension into discrete distinct parts, or breaking the original extension into distinct proper parts in imagination, and separating them at least some noticeable distance one from another; in effect, marking their subdivision by imaginatively creating spaces between them.

It should be obvious that neither of these devices has any chance of presenting the mind with an adequate idea or mental image of an infinitely divisible finite extension. If the way of marking extension by perpendicular subdividers or inserting spaces between its components is supposed to take place successively in progression over time, then no finite mind will have the infinite time needed to score the extension with infinitely many lines or spaces, even if more than one finite number of divisions can clearly be imagined to occur at a time. The reason is that lines perpendicular to and the spaces in or on the finite extension, by virtue of being particular ideas, must themselves also have some dimension, a particular width or thickness, no matter how tiny or fine, in an adequate idea or clear and distinct mental image. The subdividing markers of either type must then either be finitely extended or indivisible but constitutive of extension in aggregates of two or more. If the markers are infinite in number, as they need to be in order to score the infinite divisibility of a finite extension, then a finite extension cannot possibly accommodate them. Infinitely many perpendicular line dividers of particular width or thickness, whether finitely extended or indivisible but constitutive of finite extension in aggregates of two or more, cannot be housed within the original finite extension. Collectively by hypothesis they must constitute an infinite extension. Yet infinitely many space dividers of particular width or thickness, whether finitely extended or indivisible if

constitutive of finite extension in aggregates of two or more, must swell or expand the original finite extension to one of infinite length. And Hume emphatically denies that there can be an adequate idea or particular mental image of such an extension.¹²

The limits of the imagination in dividing extension follow from the assumption that infinitely many dividers of either type are needed to mark the infinite divisibility of a particular idea of finite extension, that two of the (extended or indivisible) dividers placed together or juxtaposed with any particular idea of any part of the finite extension constitutes a finite extension, and that arithmetically dividing an infinite number of either type of mental dividers by two leaves an infinite number remaining. The infinite number of finitely extended juxtapositions of dividers and divided parts of a particular clear and distinct idea of a finite extension in turn must comprise an infinite extension. The inference is thus directly related to, and provides another way of formulating, Hume's *reductio* from the addition of infinite parts.

It might be objected that an adequate particular idea of infinite divisibility could be manufactured by the imagination putting together first a finite line segment divided in two, and then imagining one of these halves being brought forward. Such a process is similar to the way in which a camera lens zooms in on a subject in successive stages, to be divided again in two at greater magnification, and so on, in endless repetition. The Humean reply is naturally that in order to provide an adequate idea of infinite divisibility on this model the mind must already have a concept of what it means for a process to be endlessly iterated. But to build this supposition into the objection is obviously to reason in a circle with respect to Hume's question whether the mind can have an adequate idea of infinity.

Berkeleyan Idealism in Hume's *Enquiry* Proofs

The "hint" is presented in explicitly idealist terms, stating that "all the *ideas* of quantity...cannot be infinitely divisible." But the proof is easily extended to bridge limitations of ideas about the divisibility of space to the divisibility of space itself, again by referring to the ontic requirements of adequate ideas. The result is also confirmed by the *reductio* proof against infinite divisibility reconstructed from Hume's note to Section 124. Here it is necessary only to add the assumption:

5. An adequate idea of extension and the divisibility of extension requires that extension is at most finitely divisible into extensionless indivisibles.

The possibility that Hume may allow a transition of this sort from the limitations of adequate ideas of extension to the properties of extension itself is supported in other ways by the shifts he makes from limitations of ideas of

space and time to the phenomena of space and time and back again in the *Treatise*.¹³

Hume's Berkeleyan argument in the first *Enquiry* does not explicitly state that the extensionless indivisible constituents of extension in space are sensible or experienceable rather than abstract or ideal. But in the note to *Enquiry*: 124 Hume expresses a continued commitment to sensible extensionless indivisibles, which he refers to in this place as "images" "present to the fancy or senses" as the "parts of extension" which "cannot be divided or lessened." This may suggest that in the first *Enquiry* Hume does not abandon the experienceable extensionless indivisibles of the *Treatise*, even though he does not repeat or appeal to the data of the inkspot experiment originally offered to confirm their existence.

There is a Humean argument which Hume does not explicitly offer, but which is consistent with his reflections in the note to Section 125. It indicates a way in which the proof based on Berkeley's phenomenological refutation of abstract general ideas can be expanded to establish not only that extension is not infinitely divisible and that there must exist extensionless indivisibles, but that the extensionless indivisibles are sensible or experienceable. The concept of sensible extensionless indivisibles provides a fourth alternative as a solution to Bayle's trilemma.

The continuation of the proof is given in terms of an extra assumption. The inference is placed in brackets to indicate that it is not a direct reconstruction of an argument explicit in Hume's text.

[6. If there are no abstract ideas of extensionless indivisibles, then an adequate idea of extensionless indivisibles requires that extensionless indivisibles are sensible or experienceable rather than abstract.

7. Extensionless indivisibles are not abstract but sensible or experienceable. (4,6)

8. Extension is constituted by sensible or experienceable extensionless indivisibles; space is at most finitely divisible into sensible extensionless indivisibles. (7)]

In this way, Hume's Berkeleyan argument against infinite divisibility based on the refutation of abstract ideas in the note to *Enquiry* 125 can also serve as proof in support of the positive doctrine of sensible extensionless indivisibles, and hence as an adequate later substitute for the argument Hume advances from the phenomenal experiential data of the inkspot experiment in the *Treatise*.

The Berkeleyan empiricist ground of Hume's argument against infinite divisibility in the first *Enquiry* from the nonexistence of abstract or general ideas links it indirectly to the argument against infinite divisibility based on

the phenomenal inkspot experiment in the *Treatise*. Hume was aware of Berkeley's rejection of abstract general ideas when writing the *Treatise* (T 17), but seems not to have appreciated its implications for his early critique of infinity and the theory of sensible extensionless indivisibles.

The argument hinted at by Hume in *Enquiry* 125, unlike the *reductio* proof of 124, is a new demonstration against infinite divisibility that is readily expandable to an argument that extensionless indivisibles, in a sense already implied by the opposition between infinite divisibility and extensionless indivisibility, are also sensible or experienceable. If Hume's hint is developed in this direction, it provides a complete replacement for the *Treatise* inkspot experiment and argument, and as such reflects his later thinking about the problems of extension and divisibility in the metaphysics of space and time with an argument he did not offer in the early formulation of his finitism.

Hume concludes the note to 125 by recommending the argument to scientists and mathematicians as a way of avoiding skeptical reductions to absurdity of the infinite divisibility thesis, in effect simply abandoning the thesis by invoking Berkeley's rejection of abstract general ideas and thereby in particular the idea of infinite divisibility. Simultaneously, in this way Hume gives highest approval to the Berkeleyan argument as providing the best criticism of the concept of infinity. "It certainly concerns all lovers of science," he declares, "not to expose themselves to the ridicule and contempt of the ignorant by their conclusions; and this seems the readiest solution of these difficulties" (EHU 158, note 1).

Infinity in the *Treatise* and *Enquiry*

The Berkeleyan "hint" of the second note on infinity in the *Enquiry* is in some ways the most interesting indication of Hume's later ideas about infinite divisibility and the doctrine of sensible extensionless indivisibles. The passage invites consideration of how Hume might best have completed the argument.

Hume's hint is important because it replaces the *Treatise's* inkspot experiment and argument from the experiential origin of ideas. Hume does not abandon but preserves this proposition in the first *Enquiry*. But it is worthwhile to see that his refutation of infinite divisibility and positive theory of sensible extensionless indivisibles can also be upheld without appeal to his earlier strong empiricist assumptions by an alternative route via Berkeley's repudiation of abstract general ideas. The continuities and discontinuities in Hume's thought in the period from the *Treatise* to the *Enquiry* are reflected as clearly as anywhere in the topic of infinity, in the movement from the early to the later analysis of infinite divisibility. The problem of infinity for this reason provides a useful basis for comparison.

The question of the relation between Hume's philosophy in the *Enquiry*

as compared with the *Treatise* has been repeatedly discussed by historians, and interpretations abound. There are, however, two distinct arguments against infinite divisibility in the first *Enquiry*. One of these, in the "hint," may substitute for Hume's phenomenal inkspot experiment (though it also rests finally on phenomenal considerations). Yet this, as indicated, derives more directly from Berkeley's refutation of the concept of abstract general ideas. The other is a hybrid indirect proof, that welds together elements from the inkspot experiment and the first *reductio* argument from the addition of infinite parts. If Hume devotes less space to the problem of infinity in the first *Enquiry* as opposed to the *Treatise*, it need not be because in the interim he lost interest in the problem, but rather because he has discovered a more compact and perhaps less controversial, and therefore more convincing, way of expressing the refutation of infinite divisibility.

NOTES

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1 David Hume, *An Enquiry Concerning Human Understanding* [1748], in *David Hume: Enquiries Concerning Human Understanding and Concerning the Principles of Morals*, edited by L.A. Selby-Bigge, 3rd ed. revised by P.H. Nidditch (Oxford: The Clarendon Press, 1975), 156; hereafter cited in the text as EHU.

2 Hume, *A Treatise of Human Nature* [1739–1740], edited by L.A. Selby-Bigge, 2nd ed. revised by P.H. Nidditch (Oxford: The Clarendon Press, 1978), 26–65; hereafter cited in the text as T. See Anthony Flew, "Infinite Divisibility in Hume's *Treatise*" in *Hume: A Re-Evaluation*, edited by Donald W. Livingston and James T. King (New York: Fordham University Press, 1976), 257–269; Flew, *Hume's Philosophy of Belief* (London: Routledge & Kegan Paul, 1961); Flew, *David Hume: Philosopher of Moral Science* (Oxford: Basil Blackwell, 1986); Donald M. Baxter, "Hume on Infinite Divisibility," *History of Philosophy Quarterly* 5 (1988): 133–140; James Franklin, "Achievements and Fallacies in Hume's Account of Infinite Divisibility," *Hume Studies* 20 (April 1994): 85–101; Dale Jacquette, "Kant's Second Antinomy and Hume's Theory of Extensionless Indivisibles," *Kant-Studien* 84 (1993): 38–50; Jacquette, "Hume on Infinite Divisibility and Sensible Extensionless Indivisibles," *Journal of the History of Philosophy*, forthcoming.

3 David Fate Norton, *David Hume: Common-Sense Moralist, Sceptical Metaphysician* (Princeton: Princeton University Press, 1982, rev. 1984), 239–310. See also Richard H. Popkin, *The High Road to Pyrrhonism*, edited by Richard A. Watson and James E. Force (San Diego: Austin Hill, 1980); Popkin, "David Hume: His Pyrrhonism and his Critique of Pyrrhonism," *The Philosophical Quarterly* 1 (1950); rpt., *Hume: A Collection of Critical Essays*, edited by V.C. Chappell (New York: Doubleday, 1966), 53–98; John Passmore, *Hume's Intentions*, revised ed. (New York: Basic Books, 1968), 132–151; John P. Wright, *The Sceptical Realism of David Hume* (Minneapolis: University of Minnesota Press, 1983), 92–97; Donald W. Livingston, *Hume's Philosophy of Common Life* (Chicago: University of Chicago Press, 1984); Barry Stroud, *Hume* (London: Routledge and Kegan Paul, 1977).

4 Fogelin, *Hume's Skepticism in the Treatise of Human Nature* (London: Routledge & Kegan Paul, 1985), 149–151.

5 Pierre Bayle, "Zeno of Elea," *The Dictionary Historical and Critical of Mr. Peter Bayle*, 2nd ed., edited by Pierre Des Maizeaux [1734–1738] (New York: Garland, 1984), 611–614. Hume does not explicitly mention Bayle's article in his analysis of infinite divisibility, but the historical source of his arguments and his theory of indivisibles as a response to Bayle is convincingly identified by Norman Kemp Smith in *The Philosophy of David Hume* (New York: Macmillan, 1964), 284–290, 325–338. See also Fogelin, "Hume and Berkeley on the Proofs of Infinite Divisibility," *The Philosophical Review* 97 (1988): 47–69.

6 The several reconstructions of Hume's proofs in this essay contain assumptions transcribed directly from his statement of arguments in the texts.

The only additions made are the implicit premises required for indirect proof in what appear obviously to be intended as *reductio* inferences, which Hume usually does not bother to state, and some of the subconclusions that lead from his stated assumptions to his stated conclusions by principles of elementary logic. An exception is the final argument schema of the paper. This proof is not attributed to Hume, but described only as an argument he might accept, and is accordingly placed in brackets.

7 Fogelin complains of discrepancies in Hume's use of the term "mathematical points" in *Hume's Skepticism*, 31.

8 Hume's neutrality avoids circularity in the attempt to disprove infinite divisibility by assuming that there cannot be mathematical points in the traditional abstract Euclidean sense of points, and is more in keeping with his moderate Academic skepticism in the *Enquiry*.

9 George Berkeley, *Three Dialogues Between Hylas and Philonous, The Works of George Berkeley Bishop of Cloyne*, edited by A.A. Luce and T.E. Jessop (London: Thomas Nelson & Sons, 1949), vol. II, 192–194; *A Treatise on the Principles of Human Knowledge, Works*, Vol. II, 45; Berkeley, *The Analyst or A Discourse Addressed to an Infidel Mathematician wherein it is examined whether the objects, principles, and inferences of the modern Analysis are more distinctly conceived, or more evidently deduced, than religious Mysteries and points of Faith* [1734]; *A Defense of Free-Thinking in Mathematics in answer to a pamphlet of Philalethes Cantabrigeniensis, intituled Geometry no friend to Infidelity, or a defence of Sir Isaac Newton and the British Mathematicians. Also an Appendix Concerning Mr. Walton's Vindication of the principles of fluxions against the objections contained in the Analyst; wherein it is attempted to put this controversy in such a light as that every reader may be able to judge thereof* [1735], *Works*, vol. IV, 55–102; 103–156; also "Of Infinities," *ibid.*, 233–238; Berkeley, *An Essay Towards a New Theory of Vision* [1709], *Works*, vol. I, § 54, 191: "There is a *Minimum Tangibile* and a *Minimum Visibile*, beyond which sense cannot perceive. This every one's experience will inform him." Berkeley, *A Treatise Concerning the Principles of Human Knowledge* [1710], *Works*, vol. II, § 132, 102: "If it be said that several theorems undoubtedly true, are discovered by methods in which infinitesimals are made use of, which could never have been, if their existence included a contradiction in it. I answer, that upon a thorough examination it will not be found, that in any instance it is necessary to make use of or conceive infinitesimal parts of finite lines, or even quantities less than the *minimum sensible*: nay, it will be evident this is never done, it being impossible." Compare Berkeley, *Philosophical Commentaries*, Notebook A, *Works*, vol. I Comment 441, 54: "Mem: before I have shewn the Distinction between visible & tangible extension I must not mention them as distinct, I must not mention M.T. [*minima tangibilia*] and M.V. [*minima visibilia*] but in general M.S. [*minima sensibilia*] etc." Also John Locke, *An Essay Concerning Human Understanding* [1690; 1706], edited by A.C. Fraser (Oxford: Oxford University Press, 1894), Book II, Chapter XV, § 9, 264–267. The concept of *minima sensibilia* is found in Aristotle, *On Sense and Sensibilia*, 445b33. See David Raynor's excellent essay. "'Minima Sensibilia' in Berkeley and Hume," *Dialogue* 19 (1980): 196–200.

10 Berkeley, *Principles*, 29–40. Hume endorses Berkeley's theory of representative generalization in place of abstract generalization in EHU 158,

note 1: "...all general ideas are, in reality, particular ones, attached to a general term, which recalls, upon occasion, other particular ones, that resemble, in certain circumstances, the idea, present to the mind."

11 It would not be surprising for Hume to voice skepticism about Berkeley's idealist ontology, in the claim that only minds and ideas exist, and that there are no external physical entities outside the mind. Here it would be quite appropriate for Hume to say that Berkeley's arguments "*admit of no answer and produce no conviction.*" But the note in question is attached instead to the passage in which Berkeley is credited with the refutation of abstract general ideas, a conclusion with which Hume fully concurs. The two theses are obviously distinct in Berkeley's philosophy, though the disproof of abstract general ideas plays a minor part in his defense of *esse est percipi aut posse percipere*. Hume certainly is free to accept Berkeley's repudiation of abstract general ideas without buying into the latter's idealist metaphysics. Yet here Hume raises the cry of skepticism against "all of the arguments" of the "very ingenious author" ["Dr. Berkeley"] in a comment occasioned specifically by reference to Berkeley's argument against abstract general ideas. See F.W. McConnell, "Berkeley and Skepticism" in *New Studies in Berkeley's Philosophy*, edited by Warren E. Steinkraus (New York: Holt, Rinehart and Winston, 1966), 43–58.

12 T 51: "As the ultimate standard of these figures is derived from nothing but the senses and imagination, 'tis absurd to talk of any perfection beyond what these faculties can judge of; since the true perfection of any thing consists in its conformity to its standard."

13 T 239–240: "And to cut short all disputes, the very idea of extension is copy'd from nothing but an impression, and consequently must perfectly agree to it." Hume makes parallel remarks about the immediacy of the idea of duration in time. T 36: "Five notes play'd on a flute give us the impression and idea of time..." See Jacquette, "Hume's Aesthetic Psychology of Distance, Greatness, and the Sublime," *The British Journal for the History of Philosophy*, forthcoming.

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