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Hume on Geometry and Infinite Divisibility in the *Treatise*

H. MARK PRESSMAN

Scholars have recognized that in the *Treatise* "Hume seeks to find a foundation for geometry in sense-experience."¹ In this essay, I examine to what extent Hume succeeds in his attempt to ground geometry visually. I argue that the geometry Hume describes in the *Treatise* faces a serious set of problems.

Geometric Lines

Hume maintains that ideas "are images" (T 6) which may be called up "when I shut my eyes" (T 3). "That we may fix the meaning of a word, figure," according to Hume, "we may revolve in our mind the ideas of circles, squares, parallelograms, triangles of different sizes and proportions, and may not rest on one image or idea" (T 22). As Arthur Pap notes, "'Idea' is in Hume's usage synonymous with 'mental image'." D.C.G. MacNabb also notes that "Hume thought of ideas as images, and primarily as visual images."² When Hume argues, then, that "we have the idea of indivisible points, lines and surfaces conformable to the [geometer's] definition" (T 44), he is arguing that we have mental images of geometry's points, lines and planes.

Consider geometric lines first. Geometers define a line "to be length without breadth or depth" (T 42). It is not apparent to everyone that we have mental images of such lines. According to "*L'Art de penser*" (T 43), published in 1662, it is impossible "to conceive a length without any breadth" (T 43).

We can only by an abstraction “consider the one without regarding the other” (T 43). Hume, however, offers “clear proof” (T 44), in the form of two arguments, that we actually possess ideas or mental images of geometry’s lines.

First, we have ideas or mental images of lengths with breadth, though these are supposed never to be without breadth and thus not geometric lines. (A mental image *of* length is also a mental image *with* length, just as “to say the idea of extension agrees to any thing, is to say it is extended” [T 240].) Hume reduces to absurdity the view that our mental images of length are always with breadth and thus not images of geometric lines. If our length-images were always with breadth, then our mental images would be endlessly divisible in breadth, since they would always have some positive breadth (T 43). But no mental image is endlessly divisible, for the mind arrives “at an end in the division of its ideas, nor are there any possible means of evading the evidence of this conclusion” (T 27). We thus have an idea or image of length without (divisible) breadth conforming to the geometer’s definition (T 44).

Second, we can imagine or picture the termination of a geometric plane. But a geometric plane must terminate in a geometric line (T 44). (Proof: Assume L is a line with breadth which terminates plane P. Since L has breadth, L must contain at least two parts, w and y, exactly one of which actually terminates P. But whether it is w or y, L doesn’t terminate P, which was the assumption.) Since we can picture the termination of a geometric plane, we can picture a geometric line, length without divisible breadth.

It is one thing to have ideas or mental images of geometric lines which may be called up “when I shut my eyes” (T 3). It is another for such objects actually to exist “in nature.” The conceptualist holds that though (a) geometry’s points, lines and planes are ideas, nevertheless (b) there are no such things in nature and that (c) there can’t be such things in nature:

[T]he objects of geometry... are mere ideas in the mind, and not only never did, but never can exist in nature. They never did exist; for no one will pretend to draw a line or make a surface entirely conformable to the definition: They never can exist; for we may produce demonstrations from these very ideas to prove, that they are impossible. (T 42-43)

Hume finds the conceptualist’s position inconsistent, for “whatever can be conceiv’d by a clear and distinct idea necessarily implies the possibility of existence” (T 43). That is, one cannot hold that (a) we have ideas of geometry’s objects, but that (c) geometry’s objects can’t possibly exist, for to admit (a) the mental images are actual is to admit the objects are possible. If “we have the idea of indivisible points, lines and surfaces” (which we do), then “their existence is certainly possible” (T 44).

Hume's view is that geometric lines are "apparent to the senses" (T 49): "When we draw lines upon paper...the lines run along from one point to another, that they may produce the entire impression of a curve or a right line" (T 49). Hume's position is, as F. Zabeeh notes, that "geometrical expressions such as 'points', 'lines', 'angles', etc. all denote perceptual entities."³

Straight Lines

This is not the end of Hume's "bold and original attempt" to find a foundation for geometry in visual experience.⁴ A "straight" line is a certain appearance (impression) or mental image (idea). It follows, according to Hume, that geometers cannot define a straight line without reference to these:

[M]athematicians pretend they give an exact definition of a right line, when they say, *it is the shortest way betwixt two points*. But in the first place I observe, that this is more properly the discovery of one of the properties of a right line, than a just definition of it. For I ask any one, if upon mention of a right line he thinks not immediately on such a particular appearance, and if 'tis not by accident only that he considers this property? A right line can be comprehended alone; but this definition is unintelligible without a comparison with other lines, which we conceive to be more extended. In common life 'tis establish'd as a maxim, that the streightest way is always the shortest; which wou'd be as absurd as to say, the shortest way is always the shortest, if our idea of a right line was not different from that of the shortest way betwixt two points. (T 49-50)

Hume offers two arguments to prove that "the shortest way betwixt two points" is not an adequate definition of a straight line. First, if a "streight" line were by definition "the shortest way betwixt two points," then a straight line would exist only in relation to other (longer) lines. But a straight line can exist alone, for we can picture a single straight line. So a straight line isn't *by definition* "the shortest way betwixt two points" (even if this turns out to be one of the properties of a right line).⁵

Second, the assertion "the streightest way is always the shortest" is informative. But if "streightest way" were by definition "the shortest way," then the assertion would be "the shortest way is always the shortest," which isn't informative. So "streight way" is not by definition "the shortest way."⁶ In short, any definition is, according to Hume, "fruitless, without the perception of such objects, and where we perceive such objects, we no longer stand in need of any definition" (T 637).

Hume's position is that "a right line is in reality nothing but a certain general appearance" (T 52) such as this: '_____ ' and that "we are reduc'd merely to the general appearance, as the rule by which we determine lines to be either curve or right ones" (T 49).

Geometry's Postulates

According to Euclid's 2nd straightedge Postulate, "two right lines cannot have one common segment" (T 51). Hume claims that this Postulate or "maxim" is inexact, for when compared to appearances it proves to be "not precisely true" (T 45).⁷

According to James Noxon, Hume withholds assent to the Postulate because appearances *might* disconfirm it: "One cannot be quite certain that no two straight lines have a common segment, for an inclination so slight as to be imperceptible may result in their concurring for a certain distance."⁸ Laird also claims, "If two straight lines approached at the rate of an inch in twenty leagues, who could *know*, if no one could *see*, that they did not have a common segment?" (Laird, 76).

This reading is certainly too weak. Hume does more than claim that straight lines might concur; he claims that in certain cases they *do* concur:

The original standard of a right line is in reality nothing but a certain general appearance; and 'tis evident right lines may be made to concur with each other, and yet correspond to this standard, tho' corrected by all the means either practicable or imaginable. (T 52, my emphasis)

The Postulate is true in most cases: "I do not deny, where two right lines incline upon each other with a sensible [i.e., considerable] angle, but 'tis absurd to imagine them to have a common segment" (T 51). But it isn't true in certain other cases, such as when we suppose "two lines to approach at the rate of an inch in twenty leagues" (T 51), i.e., when the two lines form an angle of about .000015°.

Hume's contemporary Thomas Reid recognized that Hume denied this Postulate. According to Reid, Hume "reasons in this manner: No man ever saw or felt a line so straight that it might not cut another, equally straight, in two or more points."⁹

Hume is right. Geometry's 2nd straightedge Postulate is false for visible straight lines which form a sufficiently small angle, no matter how thin the lines are drawn: "Every draughtsman will without doubt contend that two perpendicular straight lines factually determine a point; but this breaks down," Stefan Kulczycki notes, "for straight lines that form an angle of less than ten degrees, such straight lines do not 'determine' a point":¹⁰

Diagram A.



To vindicate the Postulate, a geometer would have to undermine Diagram A by either (i) showing that the lines in it aren't really straight, or (ii) by showing that the lines in it aren't really geometric. Hume has responded to both charges already.

The segments in Diagram A are straight insofar as they are drawn with a ruler: "Our appeal is still to the weak and fallible judgment, which we make from the appearance of the objects, and correct by a compass or common measure; and if we join the supposition of any farther correction, 'tis of such-a-one as is either useless or imaginary" (T 51). They are geometric in so far as they may be seen to be indivisible in breadth.¹¹

Points

A geometric point is defined "to be what has neither length, breadth, nor depth" (T 42). "I must ask," Hume declares in reference to geometry's points, "*What is our idea of a simple and indivisible point?* No wonder if my answer appear somewhat new, since the question itself has scarce ever yet been thought of" (T 38). Hume's somewhat new answer is that "a geometric point" (T 38) is a visible point, the smallest perceptible colored spot:

Put a spot of ink upon paper, fix your eye upon that spot, and retire to such a distance, that at last you lose sight of it; 'tis plain, that the moment before it vanish'd the image or impression was perfectly indivisible. (T 27)

When you retire from an ink spot, at a certain distance it appears indivisible in length, breadth and depth. This appearance, a visible point, is, according to Hume, geometry's point. As Norman Kemp Smith notes, "mathematical points as Hume conceives them are minima (i.e. unextended) sensibilia"¹² Visible points, or physical minima, are not original with Hume, as Harry Bracken explains:

Berkeley takes a *minimum visibile* to be that point which marks the threshold of visual acuity. Locke estimates (*Essay* II xv 9) that it is from thirty seconds to a minute "of a circle, whereof the eye is the centre," and calls it a "sensible Point." Visual *minima* constitute a sort of visual grid, I think, rather like that which appears on a television

screen. This grid is not affected by magnification glasses, since it is not characterized in terms of "dots" in the world but in terms of thresholds of sensory acuity.¹³

Does Hume maintain, as Berkeley did, that visual space (and its visible point-parts) form a two-dimensional manifold rather like a television screen, while tangible space alone admits of depth? He does.

According to Berkeley, "a man born blind, being made to see, would, at first, have no idea of distance by sight; the sun and stars, the remotest objects as well as the nearer, would all seem to be in his eye." Berkeley concludes that "one in those circumstances would judge his thumb, with which he might hide a tower or hinder its being seen, equal to that tower, or his hand, the interposition whereof might conceal the firmament from his view, equal to the firmament."¹⁴ According to Hume,

all bodies, which discover themselves to the eye, appear as if painted on a plain surface, and...their different degrees of remoteness from ourselves are discover'd more by reason than by the senses. When I hold up my hand before me, and spread my fingers, they are separated as perfectly by the blue colour of the firmament, as they cou'd be by any visible object, which I cou'd place betwixt them. (T 56)

According to Berkeley, "the ideas of space, outness, and things placed at a distance are not, strictly speaking, the object of sight" (Berkeley, Section 46), while according to Hume, "our sight informs us not of distance or outness (so to speak) immediately and without a certain reasoning and experience, as is acknowledg'd by the most rational philosophers" (T 191). Hume maintains, as Berkeley did before him, that visual space and its visible point-parts are in fact rather like a television screen, admitting of length and breadth alone.

Points, Lines and Infinite Divisibility

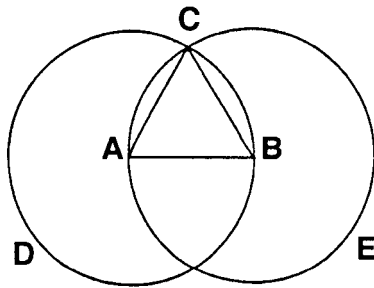
Hume insists in the *Treatise* (and in the first *Enquiry* as well) that "no finite extension can be infinitely divisible" (T 29).¹⁵ In fact Hume regards "all the mathematical arguments for infinite divisibility as utterly sophistical" (T 52). Hume's argument against the infinite divisibility of any finite extension is simply this. No finite extension can be infinitely divisible since there exist points which are both (i) part of extension and (ii) themselves perfectly indivisible.

Hume's argument is valid. No finite extension could be infinitely divisible if there were points which were both (i) part of extension and (ii) themselves perfectly indivisible. If Hume were correct that "extension is compos'd of indivisible points" (T 52), each of which is "perfectly simple and indivisible" (T 38), then he would be correct that no finite extension could be infinitely divisible. But are there such points?

Suppose the points to be geometric points. Hume offers this argument. No finite extension can be infinitely divisible since geometric points are both (i) part of extension (i.e., geometric segments) and (ii) themselves perfectly indivisible: "all the pretended demonstrations for the infinite divisibility of extension are equally sophistical; since 'tis certain these demonstrations cannot be just without proving the impossibility of mathematical points" (T 33). The argument is valid. If geometric points were part of geometric segments, there would be segments of 5 point-parts, for instance, and a segment of 5 perfectly indivisible geometric point-parts is not infinitely divisible (see Heath, 268).

The problem is that Euclid demonstrates that every geometric segment is bisectable into equal segments (Book I, Proposition 10). "We are given a uniform method," as Michael Friedman notes, "for actually constructing the point bisecting any given finite segment."¹⁶ According to Euclid's first demonstration (Book I, Proposition 1), an equilateral triangle may be constructed with any given segment as base:

Diagram B.



Given line segment AB, construct (by Postulate 3) the circles D and E with AB as radius. Let C be a point of intersection of D and E, and draw line AC and BC (by Postulate 1). Since (by the definition of a circle) $AC = AB = BC$, it follows that ABC is equilateral. Q.E.D. From this,

it suffices to join C in the proof of Proposition I.1 with its 'mirror image' below AB—the resulting straight line bisects AB (Proposition I.10). This operation...can then be iterated as many times as we wish, and infinite divisibility is thereby represented. (Friedman, 65)

Geometric points cannot, therefore, be parts of geometric segments, for if they were "it would be necessary in order to bisect the line to bisect an indivisible"

(Heath, 268), which is impossible. Consequently, since every geometric segment is bisectable into segments, “a point may be an extremity, beginning or division of a line, but is not part of it or of magnitude” (Heath, 156).

Hume thought he had caught the geometers in a contradiction. He thought that since geometric points are by definition (ii) perfectly indivisible, therefore no geometric segment could be infinitely divisible (T 42). But the faulty assumption is Hume’s, the assumption that geometric points are (i) part of geometric segments. According to Hume, “the minutest parts we can conceive are mathematical points” (T 46): “mathematical points” are such that “a certain number” of them “conjoin’d with a certain number...may make a body of twelve cubic inches” (T 239). But geometric segments are not composed of contiguous geometric point-parts like beads on a string. Thus the perfect indivisibility of a geometric point is no blow against the indefinite divisibility of a geometric segment. Isaac Barrow, whom Hume footnotes at (T 46), notes the proper relationship between geometric point and line: “if a Line be divided into six equal parts, as the whole Line answers to the Number six, so every sixth Part answers to Unity, but not to a Point, which is no Part of this Right Line.”¹⁷

Suppose instead the points in Hume’s argument to be visible points. Hume offers this argument also.¹⁸ No finite extension can be infinitely divisible since visible points are both (i) part of extension and (ii) themselves perfectly indivisible: “the table before me is alone sufficient by its view to give me the idea of extension”; “but my senses convey to me only the impressions of colour’d points” (T 34); and these colored points are “perfectly simple and indivisible” (T 38). Here the faulty assumption isn’t (i), for visible points are parts of appearances.¹⁹ Parenthetically, Hume claims here that “my senses convey to me only the impression of colour’d points” (T 34); elsewhere he claims that “nothing is observ’d but the united appearance” (T 49). He may be right on both counts according to H. H. Price: “It may both be true that we sense a complex as a whole—form-quality and all—and also true that the complex contains a finite number of sensibly distinguishable parts, which are such that no part smaller than they are could be sensed by us” (Price, 20). But now consider (ii). An appearance’s constituent visible points are “indivisible to the eye” (T 38). Hume infers that they are therefore “perfectly indivisible” (T 32) or unextended:

let us take one of those simple indivisible ideas, of which the compound one of *extension* is form’d, and separating it from all others, and considering it apart, let us form a judgment of its nature and qualities....’Tis plain it is not the idea of extension. For the idea of extension...consists of parts; and this idea...is perfectly simple and indivisible. (T 38)

Geometric points are by definition perfectly indivisible or unextended. Visible points are “indivisible to the eye” (T 38). Is being without visible parts sufficient to prove that visible points are unextended “bodies containing no void within circumference” (T 41) as Hume contends? This is the principal question.

Unextended Visibles?

Commentators have had little patience with Hume’s contention that visible points are unextended. Bracken claims that Hume contradicts himself:

Raynor and Ayers appear to interpret Hume as on the one hand insisting that the primary qualities cannot be conceived apart from the secondary, and on the other that in the case of *minima* they can be. On both points they say he is following Berkeley. I agree that Hume sometimes holds both points and is hence caught in a contradiction; I disagree that Berkeley subscribes to the second point, i.e., about unextended *minima*. (Bracken, 440)

According to Bracken, Hume holds (a) that “the primary qualities cannot be conceived apart from the secondary”—i.e., a primary quality is conceived if and only if a secondary quality is conceived, and (b) that visible points are colored but not extended. Hence Hume is caught in a contradiction. The contradiction vanishes, I submit, if we consider whether or not Hume holds (a) as Bracken alleges.

Hume maintains that visible points are not extended. He also maintains that everything extended is divisible, and that everything divisible is extended. Let’s put this compactly as: x is extended if and only if x is divisible. Hume maintains further that extension can’t be conceived apart from color: “’tis impossible to conceive extension, but as compos’d of parts, endow’d with colour or solidity” (T 228), which we may state compactly as: if x is extended, x is colored, but not as: if x is colored, x is extended. To claim all conceptions of extension are colored is to assert that if x is a (conception of) extension, x is colored; it isn’t to assert that if x is colored, x is extended. Hume, then, is committed to the following *consistent* set of claims: x is extended if and only if x is divisible; if x is extended, x is colored; and a visible point is colored but not extended.

Other commentators have other approaches. Antony Flew asserts flatly that “even a mental picture of a point must have extension.” C. D. Broad claims that “so long as I am sure that I am seeing the spot at all, I am fairly sure that the sense-datum which is its visual appearance is extended, and not literally punctiform,” and Oliver Johnson insists that “A colored point, if it is

really visible to the eye, simply cannot be one of Hume's unextended mathematical points."²¹

These commentators may be right that visible points can't be unextended, but they haven't explained what is wrong with Hume's argument to the contrary. Why isn't being "indivisible to the eye" (T 38) sufficient to prove that a visible point is "perfectly simple and indivisible" (T 38)?

"Take an inch marked upon a ruler; view it, successively, at the distance of half a foot, a foot, a foot and a half, etc., from the eye," Berkeley directs us, "at each of which, and at all the intermediate distances, the inch shall have a different visible extension, i.e. there shall be more or fewer points discerned in it" (Berkeley, Section 61). Hume agrees that more or fewer visible points will be discerned in each appearance, but denies we can determine exactly how many visible points constitute any appearance. Hume's view is that "No one will ever be able to determine by an exact numeration, that an inch has fewer points than a foot, or a foot fewer than an ell or any greater measure" (T 45) since

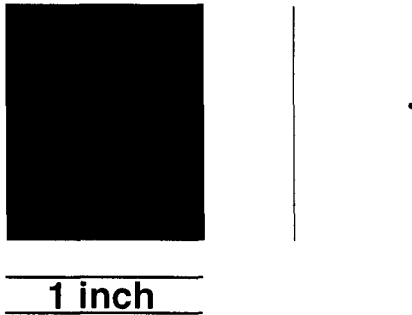
the points, which enter into the composition of any line or surface, whether perceiv'd by the sight or touch, are so minute and so confounded with each other, that 'tis utterly impossible for the mind to compute their number.... (T 45)

The problem is that Hume here overlooks calculation. We may perform the sort of "careful and exact experiments" (T xviii) Hume values to make such determinations. Retire from an ink spot until it appears indivisible to the eye. A black spot an inch in diameter appears indivisible to one with good sight and in good conditions once she has retired 500 feet from it.²²

Since there is 1 visible point to the (tangible) inch when seen from 500 feet, there are 500 visible points to the inch when it is seen from a distance of 1 foot. As James Franklin notes, there are "a mere 500 indivisibles to the inch."²³ Calculation tells us, then, that there are 500 visible points to the visible inch (an inch seen from a foot), even though we don't see each point individually. Hume himself admits that "'Tis impossible for the eye to determine the angles of a chiliagon to be equal to 1996 right angles" (T 71), though this is what the angles are equal to, and calculation determines this fact.²⁴

Now, whatever the number of visible points that constitute a given appearance, Hume denies that a single visible point more may be contained therein, for no part can "be inferior to those minute parts we conceive; and therefore cannot form a less extension by their conjunction" (T 30, note). However, retire one foot from the following segment an inch in width:

Diagram C.



Experiments show that from such an appearance (drawn taller) no more than 500 *points* indivisible to the eye can be “peeled away”—one by one—across its width. But studies also show that up to 35,000 *lines* indivisible to the eye can be peeled across the width before this appearance “becomes altogether invisible” (T 42).²⁵

Since more lines indivisible to the eye than points indivisible to the eye lie across an appearance, it follows that visible points have parts (more parts than lines indivisible to the eye). Consequently a visible point is not perfectly indivisible, as a geometric point is by definition. Hume maintains that visible points, since “indivisible to the eye” (T 38), are therefore “bodies containing no void within their circumference” (T 41). This inference fails. Being without visible parts doesn’t guarantee a lack of parts. Consequently an appearance composed of visibles “indivisible to the eye” might be infinitely divisible, despite Hume’s insistence that “no finite extension is capable of containing an infinite number of parts” (T 30). The possibility is modeled by a convergent series of fractions such as that “suggested by the formula $(1/N) \times N = 1$.”²⁶ Laird was thus correct to pose the rhetorical question: “Why should the smallest perceptible spot be other than the *minimum* perceptible *area*, and very different indeed from an unextended point?” (Laird, 69). I have simply backed this nearly universal contention with empirical data.

This experimental refutation of Hume’s argument complements John Passmore’s *a priori* analysis of why visible points can’t be unextended:

Two points can never lie contiguous to one another, because to be contiguous they would have to touch only at a certain point; and a point cannot itself touch at a point except by being that point. Hume cannot satisfactorily answer this objection. He bids us imagine a red and a blue point approaching one another. We will “evidently

perceive," he argues, "that from the union of these points there results an object, which is compounded and divisible, and may be distinguished into two parts...notwithstanding its contiguity to the other" (T 41). Now, certainly this is something we can imagine, but only because we are not thinking of Hume's [unextended] points but rather of patches of colour moving through continuous space and coming to rest alongside one another. In a strict sense, Hume is begging the question. An argument has been brought forward to show that coloured points, understood as Hume understands them, cannot lie contiguously. Hume, in reply, asserts that coloured points can be contiguous to one another: but he does not admit the proper conclusion from this fact—that points cannot be the sort of thing he takes them to be. He does not, and cannot, directly meet the argument that contiguity is impossible except between what is already complex.²⁷

Though Zabeeh claims "maybe one could ostensibly define a geometrical point" (Zabeeh, 135)—which is what Hume attempted to do—there are excellent reasons (both a *priori* and a *posteriori*) for thinking otherwise.

Geometry in the *Treatise*

Hume argues at length in the *Treatise* (and in the first *Enquiry* as well) that geometry fails to prove that any finite extension is infinitely divisible. Hume insists that "no idea of extension...consists of an infinite number of parts or inferior ideas, but of a finite number, and these simple and indivisible" (T 39). There is a smallest extension or segment—formed from two contiguous colored geometric points. "A blue and a red point may surely lie contiguous" (T 41), from which "there results an object, which is compounded and divisible" (T 41). As Kemp Smith notes, "two unextended sensibles, if contiguous, will generate what is genuinely extended!" (Kemp Smith, 300). At the same time, Hume argues that "geometry fails of evidence in this single point [in demonstrating infinite divisibility], while all its other reasonings command our fullest assent and approbation" (T 52). In short, Hume denies that every segment is divisible into segments while he endorses geometric theorems such as the Pythagorean theorem (EHU 20) and the theorem that the internal angles of a triangle equal two right angles:

relations may be divided into two classes; into such as depend entirely on the ideas, which we compare together, and such as may be chang'd without any change in the ideas. 'Tis from the idea of a triangle, that we discover the relation of equality, which its three angles bear to two right ones; and this relation is invariable, as long as our idea remains the same. (T 69)

The problem is, if segments contain some finite number of contiguous point-parts, then nearly every geometric demonstration fails, including the triangle-angle and Pythagorean theorems.

Consider what happens to the Pythagorean theorem if each side of a right triangle "consists of a finite number" of point-parts "and these simple and indivisible" (T 39). The hypotenuse of a right triangle with 100-point sides must, in Hume's system, consist of some whole number of indivisible points, perhaps 140 or 141. "For any given square," as Douglas Jesseph notes, "there will be a rational proportion between the number of minima along the diagonal and the number along the side."²⁸ The Pythagorean theorem, however, has it that if a right triangle has 100-point sides, it has a hypotenuse of 141.42135... points. Consequently, either the Pythagorean theorem (triangle-angle theorem and the rest) fail, or Hume's thesis that segments contain finitely many points fails. Jesseph, in his *Berkeley's Philosophy of Mathematics* (1993), explains:

If we accept the doctrine of the minimum sensible, most geometric theorems will be literally false...because no irrational proportion between any two finite collections of minima can be established, of any perceivable or imaginable lines or figures. (Jesseph, 77)

For example, "if the radius and circumference of a circle are both measured in terms of the number of minima which composes them, then," as Jesseph notes, "the ratio of diameter to circumference would always be expressible as a rational number" (Jesseph, 59). Franklin recognizes that this is a problem for Hume. Nevertheless Franklin claims that Hume may embrace all of geometry's conclusions while continuing to deny infinite divisibility in so far as a geometry of finitely many visible points is "indistinguishable" from Euclidean geometry:

The manner in which geometry is possible with finite numbers of points is perhaps best suggested by the calculations a computer makes in projecting an image onto its screen or to a laser printer. These have a finite resolution, so the computer...calculates with a finite precision arithmetic to decide the appropriate color for the finite number of screen pixels. Of course, the pixel itself has spatial parts but that is not "known" to the computer, which does geometry as if the pixels were simple. A computer with a very high resolution screen, with pixel size equal to the human *minimum visibile*, is a good model of what Hume thinks real space is. Like anything in computing, it is finite. The computer produces geometrical results...that are indistinguishable from those that would be produced by a geometry that incorporated infinite divisibility. (Franklin, 87)

I do not think Hume can evade inconsistency this way for two reasons. First, the sort of geometry Franklin paints, a geometry of visible points and lines, is distinguishable from Euclid's, at least according to modern geometers. Though it is true, as Adolph Grünbaum notes, that "there are certain small two-dimensional elements of visual space which are essentially isometric with the corresponding elements of Euclidean space," so that we can "obtain first-order visual approximations to the physical Euclidean geometry from viewing small diagrams frontally," nevertheless there are differences. Visual space is not, as G. Westheimer notes, "a space of constant curvature," nor "is visual space unbounded like infinite Euclidean space: the sky forms a dome." As Westheimer explains, "the many restrictive conditions which are built in during the formulation of a mathematical space cannot immediately be assumed to apply to visual space, which may, for example, be non-Archimidean." The geometry which precisely describes visual space has in fact yet to be determined: "Attempts to express the properties of visual space in formal mathematical terms (for example, postulating a hyperbolic metric for visual space) have generally not met with success."²⁹

Second (and more importantly), even if a geometry of visibles were indistinguishable from Euclid's, as Franklin suggests, *Hume* doesn't think it is. Hume argues explicitly in the *Treatise* that (parts of) Euclidean geometry are "not exact"—that some of geometry's "maxims" or postulates "are not precisely true" (T 45). Hume imputes a "defect to geometry" (T 71), since some of geometry's dictates (such as geometry's 2nd straightedge Postulate) do not match all appearances. As A. W. Moore notes, Hume regarded "geometry as an inexact science that was based on experience but misrepresented it in various ways."³⁰

Is Hume, then, simply inconsistent to hold that there are segments of 3, 4 and 5 points (for example) and to endorse such standard geometric theorems as the triangle-angle and Pythagorean theorems? The following might be offered in defense of Hume. Hume does argue that "geometry can scarce be esteem'd a perfect and infallible science" (T 71):

When geometry decides any thing concerning the proportions of quantity, we ought not to look for the utmost *precision* and exactness. None of its proofs extend so far. It takes the dimensions and proportions of figures justly; but roughly, and with some liberty. Its errors are never considerable; nor wou'd it err at all, did it not aspire to such an absolute perfection. (T 45)

Based on passages such as this one, one might construe Hume's position as follows. Segments are composed of some finite number of point-parts. The Pythagorean theorem has it that the hypotenuse of a right triangle with 100-point sides consists of 141.42135... points. Geometry is here mistaken, for

there is no such thing as a fraction of a point-part. Geometry is approximately true in such cases, but literally false. Thus infinite divisibility is denied and Euclidean geometry retained (as approximately true).

There are two problems with this defense of Hume. First, Hume doesn't consistently maintain that geometric theorems are merely approximately true. In the section "Of Knowledge," Hume apparently forgets his thesis that geometry is imprecise and claims geometry's triangle-angle theorem is "invariable" (T 69). But Euclid's Propositions are "invariable" only if every segment is invariably bisectable into two equal segments, which is not the case if every segment is composed of some whole number of contiguous point-parts. Second, this is simply not Hume's view. His view is merely that geometry is imprecise because it can never determine exactly how many whole points compose a given extension or segment, that the geometer "can never compute the number of points in any line" (T 658), and this because "the addition or removal of one of these minute [point] parts, is not discernible either in the appearance or measuring" (T 48). His view is not that "what we can measure will approximate the irrational proportions of geometry" (Jesseph, 77), for he doesn't recognize that any irrational proportions are entailed. Hume could have argued—but did not—that since every segment is composed of finitely many point-parts, it follows that any geometric conclusion which issues in a (rational or irrational) fraction of a point-part is approximately true though literally false.

Concluding Remarks

The geometry Hume presents in the *Treatise* faces a serious set of problems. Hume insists that no finite extension is infinitely divisible. "Extension" refers either to visual space (i.e., appearances) or it does not. Either way Hume's view is in trouble.

Suppose "extension" refers to visual space, which is Hume's view (T 34). Hume argues that it is impossible that any appearance is infinitely divisible, since visible points are both part of extension and themselves perfectly indivisible. This argument fails. Though "indivisible to the eye" (T 38), visible points are not "perfectly simple and indivisible" (T 38). A convergent series of fractions provides us with a model of an infinitely divisible extension, despite Hume's insistence to the contrary (T 30).

Further, if Hume's argument did succeed, if no appearance could be infinitely divisible (T 52), then Hume would be mistaken to embrace all the geometric "reasonings" he actually embraces. If Hume ties himself to a geometry of appearances that does not include infinite divisibility as a property, then he should not embrace geometric demonstrations such as the triangle-angle theorem (T 69). What is or is not demonstrable in Euclidean geometry becomes irrelevant to an accurate geometry of a finitely divisible

visual space.

Finally, if "extension" doesn't refer to visual space—if Hume is not discussing "in the eye that appearance" (T 47), then he again fails to undermine geometry's demonstration that every finite extension or segment is divisible *in infinitum*. And yet it is this demonstration and doctrine that Hume targets (in both the *Treatise* and the first *Enquiry*) as absurd and full of contradiction, being "contrary to the first and most unprejudic'd notions of mankind" (T 26).

NOTES

I gratefully acknowledge the critical suggestions of Joel Friedman, G. J. Matthey, and two anonymous referees for *Hume Studies*.

1 David Hume, *A Treatise of Human Nature*, edited by L. A. Selby-Bigge, 2nd ed. revised by P. H. Nidditch (Oxford: Clarendon Press, 1978), cited hereafter as T; Ralph W. Church, *Hume's Theory of the Understanding* (Ithaca, New York: Cornell University Press, 1935), 179.

2 Arthur Pap, *Semantics and Necessary Truth* (New Haven: Yale University Press, 1958), 75, hereafter Pap; D.C.G. MacNabb, "Hume," in *The Encyclopedia of Philosophy*, edited by Paul Edwards (New York: MacMillan and The Free Press, 1967), Volume IV, 79.

3 Farhang Zabeeh, *Hume: Precursor of Modern Empiricism* (The Hague: Martinus Nijhoff, second edition 1973), 137, hereafter Zabeeh.

4 John Laird, *Hume's Philosophy of Human Nature* (London: Archon Books, 1932; reprinted New York: Garland, 1967), 75, hereafter Laird.

5 Geometers now agree: "The notion of a straight line, owing to its simplicity, cannot be explained by any regular definition which does not introduce words already containing in themselves, by implication, the notion to be defined (such e.g. are direction, equality, uniformity or evenness or position, unswerving course)." *The Thirteen Books of Euclid's Elements*, translated with commentary by Sir Thomas L. Heath, second edition revised (New York: Dover Publications, 1956), 168, hereafter Heath.

6 "Would it be overcrediting Hume to suggest that he here anticipated at one stroke...the paradox of analysis?" (Pap, 70).

7 Rosenberg claims that Hume at (T 71) "challenged certainty about the postulate of the parallels" (Alexander Rosenberg, "Hume and the Philosophy of Science," in *The Cambridge Companion to Hume*, edited by David Fate Norton [Cambridge: Cambridge University Press, 1993], 82.) But Hume is challenging Euclid's 2nd straightedge Postulate, not the historically significant parallel Postulate (Euclid's 5th Postulate), according to which only one line parallel to a given line can be drawn through a point outside that line. Hume never mentions Euclid's 5th or parallel Postulate anywhere.

8 James Noxon, *Hume's Philosophical Development* (Oxford: Clarendon Press,

1973), 114.

9 Thomas Reid, *Inquiry into the Human Mind on the Principles of Common Sense* (1764), in *Philosophical Works*, edited by William Hamilton (Hildesheim, Germany: Georg Olms Verlagslouchhandlung, 1967), 452.

10 Stefan Kulczycki, *Non-Euclidean Geometry*, translated by Stanislaw Kanpowski (London: Macmillan, 1961), 13-14.

11 As Fogelin notes, "Hume is arguing that geometry is an empirical discipline on the grounds that it relies on observing the physical properties of diagrams" (Robert Fogelin, "Hume and Berkeley on the Proofs of Infinite Divisibility," *The Philosophical Review* 97.1 [1988], 57, hereafter Fogelin.)

12 Norman Kemp Smith, *The Philosophy of David Hume* (London: Macmillan, 1941), 297, hereafter Kemp Smith.

13 Harry Bracken, "On Some Points in Bayle, Berkeley, and Hume," *History of Philosophy Quarterly* 4.4 (1987), 437, hereafter Bracken.

14 George Berkeley, *An Essay Towards a New Theory of Vision* (1709), in *George Berkeley's Philosophical Works*. Introduction with notes by M. R. Ayers (Everyman's Library, 1975), Sections 41, 79, cited hereafter as Berkeley.

15 David Hume, *Enquiries Concerning Human Understanding and Concerning the Principles of Morals*, edited by L. A. Selby-Bigge, 3rd ed. revised by P. H. Nidditch (Oxford: Clarendon Press, 1975), hereafter EHU.

16 Michael Friedman, *Kant and the Exact Sciences* (Cambridge, Mass.: Harvard University Press, 1992), 65, hereafter Friedman.

17 Isaac Barrow, *Usefulness of Mathematical Learning Explained and Demonstrated: Being Mathematical Lectures Read in the Public Schools at the University of Cambridge by Isaac Barrow, D.D. Professor of Mathematics, and Master of the Trinity College* (1734). Translated by John Kirkby (Facsimile edition, Frank Cass: London [1970]), 48.

18 That Hume takes his points to be both geometric points and colored or visible points is well-documented in the literature. See, in addition to Kemp Smith (287) and Fogelin (32), Marina Frasca-Spada, "Some Features of Hume's Conception of Space," *Studies in the History and Philosophy of Science* 21 (1990), 401.

19 "Anyone familiar with the theories and paintings of Seurat might," as Flew observes, "characterize the Hume of this Section as 'the Father of Pointillisme'," (Antony Flew, "Infinite Divisibility in Hume's *Treatise*," in *Hume: A Re-Evaluation*, edited by Donald Livingston and James King [New York: Fordham University Press, 1976], 265).

20 H. H. Price, *Hume's Theory of the External World* (Oxford: Clarendon Press, 1940), 74.

21 Antony Flew, *David Hume, Philosopher of Moral Science* (Basil Blackwell, 1986), 43; C. D. Broad, "Hume's Doctrine of Space," Dawes Hicks Lecture on Philosophy, *Proceedings of the British Academy* 47 (1961), 166; Oliver A. Johnson, *The Mind of David Hume* (Urbana and Chicago: University of Illinois Press, 1995), 97.

22 See O. J. Grusser, "Quantitative Visual Psycholphysics During the Period

of the European Enlightenment," *Documenta Ophthalmologica* 71 (1989): 93-111.

23 James Franklin, "Achievements and Fallacies in Hume's Account of Infinite Divisibility," *Hume Studies* 20.1 (1994), 97, hereafter Franklin. Franklin puts the results, as Locke did, in terms of arc: the smallest black dot that can be seen against a white background is visible "if it occupies about 34 seconds of arc" (Franklin, 97). This way we know that one need retire only one foot from a spot of ink with diameter of .002 inches before it appears as a visible point.

24 Similarly, calculation tells us how many tangible points there are to the tangible inch. Though Hume never describes an experiment to isolate a tangible point as he does a visible point, suppose the smallest round body our finger can sense, if pressed gently, measures .001 inches in diameter. Calculation—not touch—then tells us there are 1,000 tangible points to the tangible inch.

25 See R. N. Haber and M. Hershenson, *The Psychology of Visual Perception* (New York: Holt, Rinehart and Winston, 1973); see also R. L. Gregory, *Concepts and Mechanisms of Perception* (London: Duckworth, 1974). I have not carried out the experiment in a controlled setting, but I have placed a dot a centimeter in diameter and a line a centimeter in width on paper and retired from them. I lose sight of the dot long before the line.

26 Phillip D. Cummins, "Bayle, Leibniz, Hume and Reid on Extension, Composites, and Simples," *History of Philosophy Quarterly* 7 (1990), 313-314.

27 John Passmore, *Hume's Intentions* (New York: Basic Books, 1952, revised edition 1980), 112.

28 Douglas Jesseph, *Berkeley's Philosophy of Mathematics* (Chicago: University of Chicago Press, 1993), 74.

29 Adolf Grünbaum, *Philosophical Problems of Space and Time*, in *Boston Studies in the Philosophy of Science XII*, edited by Robert Cohen and Marx Wartofsky (second edition, 1973), 157; Gerald Westheimer, "Visual Space," in *The Oxford Companion to the Mind*, edited by R. L. Gregory (New York: Oxford University Press, 1987), 796.

30 A. W. Moore, *The Infinite* (London: Routledge, 1990), 82.

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