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Infinite Divisibility and Actual Parts in Hume's *Treatise*

THOMAS HOLDEN

I believe that the smallest portion of matter may be practically divided *ad infinitum*; that equal qualities taken from equal qualities, an unequal quality will remain; that two and two make seven; that the sun rules the night, the stars the day; and the moon is made of green cheese.

Tobias Smollett, *The History and Adventures of an Atom* (1769)¹

Introduction

The ferocious controversy in early modern natural philosophy over the structure of continua focuses on a cluster of supposed paradoxes of infinite divisibility. According to the recent commentary, these paradoxes are straightforwardly mathematical in nature: they simply challenge *mathematical constructions of infinite divisibility*. So interpreted, the paradoxes are then quite easy to disarm. They rest on quaint mathematical mistakes—forgivable in the early modern period, perhaps, but clear errors all the same.

But this interpretation of the Enlightenment controversy will not stand scrutiny. The early modern debate depends crucially on a body of *metaphysical* doctrine concerning the 'filling' or 'stuffing' of actual physical continua—a body of doctrine that dominates the natural philosophy of the period and that sets the background for the debate over infinite divisibility. The controversy is

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not then a purely mathematical debate, exclusively concerned with the mathematically tractable properties of continua. And once we appreciate this, we will see that the paradoxes are not so readily dismissed.

In this paper I focus on Hume's main argument against infinite divisibility in Book 1, part 2 of the *Treatise*.² Using this argument as an example, I want to embarrass the standard mathematical reading and vindicate my alternative metaphysical interpretation. The point can then be generalized to the wider Enlightenment debate. I focus on Hume's argument in particular, since here there is a wealth of commentary and an exceptionally clear case of a major Enlightenment philosopher charged—quite unjustly, I will argue—with the most grotesque mathematical blundering.

I. Hume's Lead Argument against Infinite Divisibility and the Standard Objections

Although there are several arguments against infinite divisibility in the notoriously thorny Book 1, part 2 of the *Treatise*, commentators have—naturally enough—focussed on the one argument that heads his discussion and that Hume clearly sees as the centerpiece of his case.³ In sections 1 and 2, Hume frames this lead argument in terms of the divisibility of our *ideas* and *impressions* of extended entities. But in section 4, he makes it clear that similar reasoning would apply to extended entities in the extra-mental physical world.⁴ And—as has been well pointed out in the literature⁵—the lead argument is indeed general: it purports to show that *no* finite thing can be infinitely divisible, and that *every* finite thing must resolve to a finite number of first elements. So we can follow the commentators in bracketing the fact that Hume introduces his argument in terms of mental representations of extended things. It applies no less to extended entities in the physical world.

Hume's lead argument is brisk. It runs as follows:

(H1) “[W]hatever is capable of being divided *in infinitum*, must consist of an infinite number of parts”; “Every thing capable of being infinitely divided contains an infinite number of parts” (T 1.2.1.2, 1.2.2.2; SBN 26, 29).

(H2) “[T]he idea of an infinite number of parts is individually the same idea with that of an infinite extension; . . . no finite extension is capable of containing an infinite number of parts” (T 1.2.2.2; SBN 30).

Therefore: (H3) “[N]o finite extension is infinitely divisible” (T 1.2.2.2; SBN 30).

In short: (i) whatever is infinitely divisible has an infinite number of parts; (ii) whatever has an infinite number of parts is infinitely large; so (iii) nothing finitely extended is infinitely divisible. I will be maintaining that this argument is essentially incomplete or abbreviated. Were it stated in full, certain metaphysical background assumptions that are currently implicit would become explicit. But for the moment let us put my interpretation to one side and entertain the standard reading: (H1)–(H3) is not a metaphysical enthymeme, but rather a *complete* mathematical argument against the *mathematical* possibility of infinitely divisible quantity.

So interpreted, argument (H1)–(H3) seems to exhibit a variety of flaws. And it is here that the commentators—taking Hume’s premises as purely formal, mathematical principles about the geometrical properties of the extended—have been quick to find fault with Hume’s elementary mathematical abilities.

First, premise (H1)—the principle that whatever is infinitely divisible has an infinite number of parts—is woefully misguided. Antony Flew writes that this “ruinous premise” is “mistaken twice over” and “without qualification false”; Robert Fogelin, that it amounts to “a conceptual confusion”; Marina Frasca-Spada, that there are “many excellent reasons for saying that [(H1)] is entirely mistaken.”⁶ Flew states the “absolutely crucial” objection to (H1) clearly:

[T]o say that something may be divided *in infinitum* is not to say that it can be divided into an infinite number of parts. It is rather to say that it can be divided, and sub-divided, and sub-divided as often as anyone wishes: infinitely, without limit. That this is so is part of what is meant by saying: ‘Infinity is not a number!’⁷

Infinite divisibility, Flew insists, just involves the ability to endlessly divide and subdivide: it decidedly does *not* involve the ability to divide something into a greater-than-finite number of parts. In short, it requires only a *potential infinity* of parts (an ever-increasable but always actually finite number of parts) not an *actual infinity* (a completely given greater-than-finite number).

Now, in addition to this first objection to (H1), Flew adds a second criticism that he thinks “less important”:

[To] say that something is divisible into so many parts is not to say that it consists of—that it is, so to speak, already divided into—that number of parts. A cake may be divisible into many different numbers of equal slices without its thereby consisting in, through already having been divided into, any particular number of such slices.”⁸

This is interesting, since here Flew is brushing up against a substantive metaphysical issue concerning the ontological status of parts that was hotly contested in the early modern period and that will set the framework for my interpretation and defense of Hume (in section 3 below). So Flew does seem to be at least dimly aware that there are metaphysical issues in the background of (H1)–(H3), and that it is not simply an argument within mathematics. Notice, however, that his objection is underdeveloped: he seems to think that Hume’s way of conceptualizing division and parthood is just obviously wrong, and that the cake example shows this.⁹ Moreover, he continues to insist that the “crucial” criticism of (H1) is not this second “less important” point, but rather the former, mathematical objection, shared by Fogelin and Frasca-Spada. Flew is still thinking of Hume’s argument as fundamentally a mathematical one, just as they do.

Now to premise (H2): the principle that whatever consists of an infinite number of parts must be infinitely large. Commentators have also singled out this premise as mathematically mistaken. Certainly something that consists of an infinite number of *same-sized* parts will be infinitely large. But if those parts are of proportionately diminishing size, then the whole need not be infinite in size. Fogelin presses this objection to (H2) as follows:

It is true that if we take a finite extension (however small) and repeat it *ad infinitum*, we will get an infinite extension. That, however, is quite beside the point, because the argument for infinite divisibility depends on the possibility of constructing ever smaller finite extensions, as in the sequence [1/2, 1/4, 1/8, etc.] whose sum approaches, but does not exceed, 1.¹⁰

Now, Hume does mention this objection in a footnote to his argument, but promptly dismisses the distinction between ‘proportional’ parts and ‘aliquot’ (same-sized) parts as “entirely frivolous.”¹¹ But most commentators have been inclined to think that this distinction is not half so frivolous as Hume claims, and that the current objection to (H2) is a good one.¹²

We have, then, two distinct mathematical objections to Hume’s argument. Contra (H1), whatever is infinitely divisible need *not* consist of an infinite number of parts; and, contra (H2), something that consists of an infinite number of parts need *not* be infinite in size. In addition to these two particular objections—each of which identifies a *specific* mathematical mistake—we might also add the general objection that, since there certainly is a mathematically coherent conception of infinite divisibility, there must be a mistake *somewhere* in Hume’s complaint against it. James Franklin puts this last general objection as follows:

The infinite divisibility of space and time is possible. (This is because there is a consistent model that incorporates infinite divisibility, namely the set of infinite decimals.) It follows that all supposed proofs of the impossibility of infinite divisibility, whether mathematical or philosophical, are invalid.¹³

II. The Metaphysical Background to Hume's Lead Argument

Hume has not been treated fairly by these commentators, nor is his lead argument against infinite divisibility so easily overturned. The commentators' central mistake is that they interpret Hume to be raising a purely *mathematical* challenge to the notion of infinitely divisible quantity in general. It is this interpretation that leads to the obvious mathematical objections reviewed above. But this reading is quite mistaken. Once we set Hume's argument against the early modern debate concerning the ontological status of the parts of physical continua, we will see that it does not challenge mathematical constructions of infinite divisibility *in abstracto*. Rather it sets out an objection to the infinite divisibility of *physical quantities with concrete, actual parts*. Once the argument is read this way, the stock mathematical objections all miss their target, as I will show in the next section. But first I must introduce a controversy in early modern metaphysics that sets the framework for a proper understanding of Hume's argument. This is the clash between the actual parts and potential parts doctrines—two rival metaphysical theses concerning the parts of physical continua. Are such parts fully-fledged concrete existents? Or are they merely *possibilities* or *potentialities* until actualized by a positive operation of division?

The Actual Parts Doctrine

According to this first doctrine, the parts of a given physical continuant—a body, for instance—are each a distinct existent. They each exist independently of the whole, and independently of all the other non-overlapping parts, and they each exist prior to any positive act of division. The parts are all already embedded in the architecture of the whole: division merely separates or unveils them, it does not create them anew. The whole continuant is thus an aggregate: a composite of so many distinct parts, so many independent beings. (Any given part may of course be dependent on the parts composing it—but it will be independent of the whole, and independent of those parts that it does not partially or wholly overlap.)

It is important to be clear that the actual parts doctrine states a thesis about the *ontological* or *metaphysical* status of the parts of continua. The doctrine must not be confused with the view that continua have parts that are differentiated by any heterogeneous *physical organization*. For all that is said here, continua such as bodies might be perfectly homogeneous in their internal physical organization or architecture, altogether lacking *physically* differentiated subparts. Think, for instance, of a block of jelly that is, so to speak, homogenous jelly-gloop all the way down. According to the current doctrine, such a block of jelly is still a composite of so many ontologically independent parts, its physical homogeneity notwithstanding. Or, to give the classic example: on the actual parts view, Michelangelo did not, properly speaking, create his *David*. This particular shapely block of marble existed before Michelangelo laid hands on his chisel—as did its subparts and indeed numerous other perfect overlapping copies. It is simply that Michelangelo’s genius enabled him to bring this particular block to light—rather than, say, one of the less attractive blocks that also existed.

At least as an analysis of the structure of *matter* (though not of space) the actual parts account corresponds to the popular position in contemporary metaphysics now generally known as the doctrine of arbitrary undetached parts: the undetached parts of any material body each exist as so many distinct beings.¹⁴ And, thus restricted to an analysis of matter, the actual parts account was certainly overwhelmingly popular with the partisans of the new science of the Enlightenment. Here, for instance, is an endorsement of the actual parts analysis of matter from Samuel Clarke in 1707:

some of the *first* and *most obvious* Properties which we *certainly know of Matter*, [include] its having partes extra partes, strictly and properly speaking, that is, its consisting of such Parts as are *actually unconnected*, and are *truly distinct Beings*, and *can* (as we see by Experience) *exist separately*, and *have no dependance one upon another*.¹⁵

And, similarly, Thomas Reid writes in 1785 that:

There seems to be nothing more evident than that all bodies must consist of parts, and that every part of a body is a body, and a distinct being, which may exist without the other parts. . . . when [matter] is divided into parts, every part is a being or substance distinct from all the other parts, and was so even before the division.¹⁶

In fact Clarke’s and Reid’s endorsements of the actual parts analysis of matter are merely the tip of an iceberg. Virtually all the new philosophers of

this period ratify this account of material body—including, for instance, Descartes and the Cartesians,¹⁷ Walter Charleton,¹⁸ Isaac Barrow,¹⁹ Ralph Cudworth,²⁰ Pierre Bayle,²¹ Leibniz,²² and Newton and the Newtonians.²³ As we shall see, Hume also sponsors the actual parts analysis of matter and indeed an actual parts analysis of *all* continuous quantities.

Why was the actual parts analysis of matter so overwhelmingly popular in the early modern period? In fact, explicit arguments for the doctrine turn out to be surprisingly rare. For most of these early modern philosophers, the account simply had axiomatic status: it was an unargued (and often implicit or even unconscious) background presupposition rather than a contentious doctrine requiring defense. But, while this paper is not the place for a full presentation and assessment of the arguments, we can in passing point toward some of the main lines of thought that seem to stand behind the doctrine's popularity, and which are presented explicitly in some early modern writings at least.²⁴ First, divisibility was thought to (conceptually) presuppose the prior existence of logically distinct parts: an entity could only be divided if it already contained so many distinct parts. Second, the parts post-division are clearly so many distinct beings; but the parts post-division are surely the self-same entities they were prior to division; thus the pre-division parts must already be so many distinct beings. Third, diverse parts occupy distinct places; but whatever occupies a distinct place is a distinct thing. Fourth, diverse parts can support contradictory properties; they must thus be conceptualized as so many distinct substances. To these (perhaps somewhat unimpressive) particular arguments, we can also point to the more general way in which the actual parts metaphysic would have resonated with the new science's demand for explanation in terms of pre-formed physical structures and the corresponding suspicion of the scholastics' emphasis on potentiality. Similarly, the doctrine fits well with the new philosophers' insistence on the ontological priority of relata over relation—here, the ontological priority of constituent part over composed whole. This is in opposition to the scholastic doctrine of substantial forms which asserts the co-dependence of at least some relations and their relata: the identity of the parts of substances—the relata—being dependent on their functional role defined by the relation in which they stand to one another, that is, by the whole form.

The Potential Parts Doctrine

According to the rival potential parts doctrine, the parts of a given continuant (such as a body) are not distinct existents prior to their being actualized by a positive operation of division. Rather division *creates* these parts anew—it does not simply separate pre-existing parts. Prior to the act of division that

generates these parts, the whole is best described as containing *possible* or *potential* parts. But such 'potential parts' simply represent ways in which the whole could be broken down, and talk of potential parts is really just talk about the modal properties of the whole original. Suppose, for instance, that a body is divisible into four parts. The potential parts theorist may then concede that it contains four potential parts. But (properly understood) this is not to concede the existence of four shadowy pseudo-entities. Rather it is just to say of the whole—the one thing that is there to be counted—that it has the power or the ability to be divided into four distinct parts, thereby creating four new beings. On this view, then, the whole is not a composite or aggregate structure, a construction from so many distinct actual parts. It has no particular inherent structure of ontologically distinct parts prior to division.²⁵

As with the rival actual parts doctrine, it is important to appreciate that this is again a *metaphysical* thesis about the ontological status of the parts of continua, not a thesis about their internal *physical* architecture. For all that the current doctrine asserts, a given continuous entity with potential parts might be either completely homogeneous or highly complex in terms of its internal physical structure or organization.

The potential parts doctrine is central to Aristotelian natural philosophy. Ratified by St. Thomas, it becomes the orthodoxy of high medieval and Renaissance scholasticism. One or two of the new philosophers (such as Hobbes) also endorse it—though it is important to appreciate that on this issue they stand opposed to the overwhelming majority of the Enlightenment's new anti-scholastic philosophers.²⁶

As with the actual parts account, we can again briskly gesture toward the main lines of thought behind the potential parts system. (This is not the place for a full presentation and assessment of the arguments, however.) First, some potential parts advocates maintained that, as a conceptual matter, division must be understood as the *creation* of several entities from one. This would seem to entail that the parts pre-division are simply potentialities rather than distinct beings prior to their subsequent actualization through division. Second (and importantly for the purposes of this paper, as we shall see shortly), the potential parts doctrine seems able to accommodate infinite divisibility without admitting an actual infinity of pre-existing parts, and thus avoids the paradoxes that supposedly follow from such an admission. Third, the potential parts metaphysic would have been conducive to those sympathetic to scholastic natural philosophy, according to which the basic unit of the material realm is an Aristotelian substance—a quantity of matter organized by a particular form (paradigmatically, an organic form). Such a substance is a genuinely unified whole, not a composite or aggregate of the parts into

which it may be divided. Prior to division, these parts do not exist independently: since their identity is determined by their functional role in the whole substance, they are merely aspects of features of that one whole. Only when they are divided are the parts then actualized as so many distinct beings.²⁷

III. The Actual Parts Reading of Hume's Lead Argument

Armed with our new understanding of the rival actual parts and potential parts doctrines, we are now ready to return to the defense of Hume's lead argument (H1)–(H3). First recall that, according to the standard reading found in the secondary literature, Hume's argument is supposed to challenge *mathematical constructions* of infinitely divisible continua. And, as we saw, this purely mathematical interpretation of the argument leads directly to the obvious mathematical objections raised by the various commentators. But now we are in a position to present a different interpretation of Hume's lead argument. On my new reading, the argument is not supposed to raise a formal problem for mathematical models of infinite divisibility, divorced from all thought of the stuffing or filling of actual physical continua. Rather (on my interpretation) *the argument must be set in the context of the actual parts account*—an account that was generally accepted as an analysis of matter in Hume's day, and that was accepted by Hume as an analysis of *all* continuous quantities (as we shall see below). On this reading, the argument raises an objection to the infinite divisibility of *physical quantities with actual parts*—not a purely mathematical challenge to infinitely divisible quantity *in abstracto*.

I will substantiate Hume's commitment to the actual parts metaphysic and provide further evidence for this reading in a moment. But first let us look at the way this interpretation disarms the standard mathematical objections.

As the reader will recall, those objections were as follows. *First*, (H1) was rejected, since infinite divisibility required only a potential infinity of parts, not an actual infinity. *Second*, (H2) was rejected, since an infinite number of parts need not sum to form an infinitely large whole. *Third*, there was the general objection that, since there certainly is a coherent mathematical model of infinite divisibility, there must be a mistake somewhere in Hume's reasoning. We can deal with each of these in turn.

First, given my actual parts interpretation, the objection to (H1) immediately evaporates, since infinitely divisible continua with actual parts must indeed each have an actual infinity (a greater-than-finite number) of parts pre-given. According to the actual parts doctrine, each of the parts into which the whole can be divided *already* exists prior to division. So if a continuum can be divided without end, it cannot merely have an indefinite potential

infinity of parts; it must already have a greater-than-finite number of them laid up in it from the very start.²⁸ (Here the obvious contrast is with the potential parts metaphysic, where parts are created anew as division proceeds. On the potential parts account, infinite divisibility *does* only entail an indefinite potential infinity of parts.)

Second, the objection to (H2) is much too quick. If a continuant, such as a body, has actual parts, then it is a composite or compound entity: a structured aggregate of so many distinct parts. And, at least according to the standard orthodoxy of the early modern period, such a composite structure will depend for its existence on the ontologically prior existence of its parts. If these parts are themselves also composite, then they will in turn depend for their existence on the ontologically prior existence of their subparts, and so on. But (according to this traditional argument) given that the whole original composite structure exists, the regress of ontological dependence set off here cannot go on forever without a ground floor. There must then be an ultimate bedrock of noncomposite first parts from which the whole original ultimately derives its existence. So continua with actual parts must have noncomposite, *ultimate* parts: atomic elements that, assuming their homogeneity, will be *aliquot* (same sized) *simples*.²⁹ Given the actual parts metaphysic, then, if one also accepts the ontological regress argument to simple first parts and the uniformity of those ultimate simples, the proportional parts objection to (H2) is indeed as “entirely frivolous” as Hume insists. And, once again, we can note the contrast with the potential parts metaphysic, where the proportional parts objection to (H2) would make perfect sense.³⁰ (Hume presents this “very strong and beautiful” ontological regress argument from composites to simples at T 1.2.2.3 [SBN 30], where it is clearly intended to support (H2). It is, of course, a famous pattern of reasoning—most familiar, perhaps, from the opening sections of Leibniz’s *Principles of Nature and Grace* and *Monadology*, or from Kant’s Second Antinomy in the *Critique of Pure Reason*.³¹)

Third, the general objection of Franklin also misfires once Hume’s argument is placed in its proper, actual parts context. If there is a coherent mathematical model of infinite divisibility, this merely shows that there can be no purely *formal* or *mathematical* complaint against it. It certainly does not show that there could be no *metaphysical* complaint against that model being translated into an actually existing physical structure.

With each of the three standard mathematical objections then, the response is at bottom the same: Hume’s argument is not properly criticized by considering abstract mathematical constructions of infinite divisibility. The standard objections that do this all overlook the actual parts context of the argument and are thus far misdirected.

In fact, of all the objections reviewed above, the only one that remains on target is Flew's "less important" criticism of (H1)—actually, an objection of the utmost importance. The objection was that physical continua need not contain as many parts as they can be divided into. This constitutes an outright rejection of the actual parts doctrine, and of course this *would* constitute a direct challenge to Hume. Given my metaphysical reading, Hume's argument depends crucially on the actual parts doctrine and will collapse if that doctrine is false. However, this does not compromise *my* argument that Hume's case against infinite divisibility is a metaphysical one that presupposes the actual parts doctrine; in fact, of course, it underlines it. (Notice also that Flew offers no real argument against the actual parts view. He simply takes the cake example to establish the potential parts account—but of course this simply begs the question against the actual parts advocate, who will maintain that the slicing of the cake does not involve the literal creation of new parts, but rather the separation of pre-existing parts.)

We can then see why Hume would take the standard objections to fall short once his argument is placed in the context of the actual parts metaphysic and is understood as an attack on the suggestion that material bodies (or, indeed, any continuous quantity with distinct, actual parts) could be divisible *in infinitum*. This gives us one reason to think that my metaphysical reading of Hume's case against infinite divisibility is correct, and that the orthodox mathematical reading in the secondary literature is mistaken. But perhaps it will be insisted, on behalf of the mathematical reading, that Hume's text does *not* explicitly set out the supposed actual parts context of his reasoning. And this may seem to support the purely mathematical interpretation of the argument. At the very least, it may seem to *allow* that interpretation.

If I am right that Hume's argument is intended to apply only to physical quantities with distinct, actual parts, it certainly *must* be confessed that Hume failed to make this explicit. However, there are a number of supporting considerations that bolster my interpretation and cause further trouble for the purely mathematical reading.

First, we should stress just how appalling Hume's argument would have to be given the mathematical interpretation. On this reading, Hume stands charged with failing to appreciate certain absolutely elementary facts about fractions: for instance, that a proportional division such as $1/2$, $1/4$, $1/8$, $1/16$, etc. . . ., can be iterated as many times as one likes but its sum at any point will always be less than (though ever closer to) a finite limit. (Notice here that, whatever one thinks of the period's travails over fluxions, infinitesimals, and the problematic notion of a *completed* infinite series, this basic fact about *potentially* infinite series of proportional divisions was widely understood long before Hume's time. The elementary point here is in Eudoxus,

Aristotle, and Euclid, and it was pressed by scholastics over and over during the centuries before the calculus, for instance.)

Second, to treat Hume's argument as a purely mathematical one divorced from the actual parts metaphysic is to fail to appreciate its connection to the neo-Epicurean tradition from which it is drawn. In the hundred years or so before Hume, the self-same argument is sponsored by Henry More, Joseph Glanvill, the younger Isaac Newton, and George Berkeley, and is reported by Isaac Barrow and John Keill.³² Moreover—and most importantly for our purposes—it is also presented by Kenelm Digby, Walter Charleton, and Pierre Bayle, each of whom gives the abbreviated enthymeme captured by (H1)–(H3), *and then sets it in the actual parts context*, expressly identifying its additional metaphysical premises.³³

Charleton's text (the *Physiologia* of 1654) is especially gratifying on this count, since he first sets out the abbreviated version,³⁴ and then turns to address what he calls the traditional Aristotelian and Stoic objections. And these turn out to be precisely the same mathematical complaints pressed against (H1) and (H2) by Flew, Fogelin *et al.* in the recent literature: first, contra (H1), "that the division of finite body into infinite parts doth not make it actually infinite, because the parts are not actually, but only potentially infinite"; and, second, contra (H2), that "by admitting an infinity of parts in a Finite Continuum a Continuum doth not become infinite; because that results properly not from *Proportional*, but *Aliquotal* parts."³⁵ Charleton's response is that each of these objections is easily blocked by the invocation of the actual parts doctrine. Once the argument is clearly placed in its actual parts context, it is rescued from "the Sophistry of the most specious Recesses invented to assist the Contrary opinion" and stands as "perfectly Apodictical and . . . inoppugnably victorious."³⁶ Setting aside Charleton's lavish prose, the moral here is clear. Hume's reasoning must be placed in a tradition in which the argument is invoked over and over. And in that tradition it is clear that the argument is understood to rest on the actual parts doctrine.

Third, the stock mathematical objections so popular in the recent literature—and reported by Charleton—would certainly have been familiar to Hume. Aside from the fact that the argument was traditional and the objections to it generally well-known, we know that Hume was definitely acquainted with Barrow's *Mathematical Lectures* and Bayle's famous article on Zeno of Elea in the *Historical and Critical Dictionary*. It is also extremely likely that he was familiar with Keill's highly influential *Introduction to Natural Philosophy*.³⁷ Now, in Barrow's text and again in Keill's, we find each of the two standard mathematical objections clearly set out. And in Bayle's, we find at least the objection to (H1) addressed.³⁸ So Hume would certainly have been aware of the stock mathematical objections deployed by Flew, Fogelin, *et al.*

This, once again, bolsters the suggestion that Hume could not have been guilty of such obvious errors, and that more is going on in his argument than the standard mathematical reading allows.

Fourth, as was noted above, Hume *does* explicitly address the stock objection to (H2) in one of his rare footnotes. He suggests that, at least for the purposes of this argument, the distinction between proportional and aliquot parts is “entirely frivolous.” As we have seen, my metaphysical reading makes good sense of this dismissive attitude. On the other hand, the mathematical reading can make no sense of this. The commentators have had to claim that Hume’s offhanded insouciance here is simply unjustified. But they can offer no plausible explanation of *why* Hume thought the distinction frivolous.

Fifth, and finally, we should note that Hume does clearly subscribe to an actual parts analysis of all continua. For instance he writes that “A real extension . . . can never exist without parts, different from each other.”³⁹ This directly follows of course from Hume’s more general principle that *all* entities that are separable or distinguishable are distinct existents: “whatever objects are separable are also distinguishable, and . . . whatever objects are distinguishable are also different.”⁴⁰ And this is simply one direction of the famous biconditional known as Hume’s Separability Principle. Such a principle thus incorporates a commitment to actual parts. Hume’s argument must thus be understood in this metaphysical context, in which continua are understood to have distinct, actual parts.

IV. The Actual Parts Reading and Hume’s Atomist Phenomenology

Before concluding, I must mention one particularly perceptive commentator who stands apart from the standard mathematical interpretation. Rosemary Newman has clearly noted that Hume’s argument turns on underlying principles that the standard mathematical interpretation overlooks:

Hume . . . is asserting that the idea of a potential infinity cannot suffice to support the implications of the thesis of infinite divisibility. The nature of the alleged deficiency is laid out in Principle [(H1)], which reveals Hume to be of the opinion that the concept of infinite divisibility has application solely to composites whose parts number an actual infinity. From this it follows that infinite divisibility must presuppose the concept of an actual infinite number and cannot be coherently conceived independently of the coherence of that concept.⁴¹

All this is exactly right. Newman has seen what most commentators have missed: that Hume is aware of the claim (of Flew, Fogelin, *et al.*, along with Barrow, Keill, the Stoics, and the Aristotelians) that infinite divisibility requires only a potential infinity of parts, and that he believes he has grounds for rejecting this for the view that infinite divisibility requires an actual infinity of parts pre-given. However, I do not agree with Newman's explanation of *why* Hume thought this was the right way of thinking of infinite divisibility. Having admitted that this is something of a puzzle, Newman then proposes the following explanation:

[T]he reason . . . for his adoption of Principle [(H1)] . . . arises out of the atomistic phenomenology advanced in the first Part of Book 1 of the *Treatise*. Here Hume holds simple ideas to be the ultimate elements from which knowledge of reality is constructed. . . .

Given this atomist conception of reality as a construction out of simple impressions and ideas, Principle [(H1)] becomes a necessary truth. Division of any idea by the imagination cannot be other than the analysis or separation of a complex idea into its component simple ideas. . . .

The same atomistic conception of reality also renders redundant any suggestion of a distinction between aliquot and proportional parts of a whole. For Hume, the concept of a continuous magnitude is itself ruled out by the character of his epistemology.⁴²

In short, Hume's phenomenological atomism—the famous doctrine of *minima sensibilia*, indivisible pixels that concatenate, in finite arrays, to build our visual and imaginative fields—both guarantees (H1) and blocks the proportional parts objection to (H2). Now, I certainly agree that Hume does subscribe to this sort of atomistic phenomenology, and I agree that this rules out the infinite divisibility of impressions and ideas of extended objects. I also agree that—elsewhere in the *Treatise*—Hume does explicitly advance this sort of phenomenological argument against the infinite divisibility of our *ideas* and *impressions* (see especially T 1.2.1.3 and T 1.2.3.12–15; SBN 27 and 38–9). But as a reading of Hume's lead argument, this cannot be quite the full story.

First, although Hume does indeed allude to the doctrine of *minima sensibilia* when introducing the lead argument (H1)–(H3), his argument does not in fact depend on that doctrine. As we have seen, Hume gives us an *argument* from actual parts to simple first parts (the “very strong and beautiful” ontological regress argument)—an argument, notice, that would apply to *any* entity with actual parts. He does not simply start out with a doctrine of simple first parts.

Second, it misses the point (rightly stressed by Flew and Frasca-Spada) that Hume intends his argument to be entirely general. It purports to show that *no* continuous quantity (or at least no continuous quantity with distinct, actual parts) could *possibly* be infinitely divisible; not just that our ideas and impressions, as experience happens to show us, are only finitely divisible. It is in this sense an *a priori* argument rather than an *a posteriori* argument resting on introspection. In particular, Hume's argument expressly applies not only to the internal realm of ideas, but also to extended things out there in the real world.⁴³

Third—and absolutely crucially—if Hume's argument (H1)–(H3) did indeed rest on his prior commitment to the thesis that whatever is extended is made up of an array of simple first elements, then it would be entirely question-begging. This is precisely the sort of claim that Hume's opponents on the question of infinite divisibility expressly rejected. So he can scarcely assume that extended things must be collections of such simples in order to refute infinite divisibility: this would be a manifest *petitio*. And here we can contrast my interpretation, according to which it is the more fundamental actual parts doctrine that underpins Hume's reasoning. This doctrine was almost universally accepted during Hume's period, even by proponents of infinite divisibility. So Hume's appeal to this metaphysical doctrine does not make his argument a *petitio*. His argument is rather that if one understands a given extended entity to have distinct, actual parts (as most of his opponents would accept is true of material objects at least), then it cannot be infinitely divisible. Hume's lead argument is intended to *establish* (from premises his dialectical foes accept) that divisibility can be only finite; he is not simply *presupposing* it by starting out with his own idiosyncratic doctrine of minimal parts. Newman certainly deserves credit for noticing that Hume's argument is not purely mathematical and that there are metaphysical doctrines lurking in the background. But in focusing on a particular doctrine that is quite peculiar to Hume, she misses the lead argument's deeper roots in the actual parts metaphysic and its heritage in the neo-Epicurean tradition of Charleton *et al.*, and takes Hume to be advancing a demonstration whose premises are an anathema to his dialectical foes. Her failure to appreciate this point is certainly underlined by her claim that, absent an appeal to Hume's phenomenological atomism, Flew's mathematical objections to (H1)–(H3) are correct.⁴⁴ (This, of course, misses the point that Hume can invoke the actual parts doctrine to resist the various obvious mathematical objections.)

Conclusion

I hope I have cleared Hume in the face of the various charges of gross mathematical incompetence that stem from the standard mathematical

interpretation. Here at least he deserves an acquittal. (Whether or not his lead argument is ultimately successful is of course another question.) But it seems to me that this lesson can be generalized beyond the confines of Hume interpretation to apply to the wider Enlightenment debate over continua and infinite divisibility. Such an extension of the actual parts interpretation would take us beyond the limits of this paper. But it is at least worth noting that much of the recent literature on the early modern debate presents a purely mathematical reading, rather than a reading that is sensitive to the background role played by the actual parts-potential parts controversy. (This is, I suspect, because the metaphysical framework of the debate only comes fully into focus when one examines certain less well-known seventeenth-century texts such as Digby's and Charleton's, in which the actual parts-potential parts controversy is still very much alive. By the eighteenth century, the actual parts doctrine has become axiomatic for most philosophers and is often simply implicit and perhaps even unconsciously assumed in their arguments concerning infinite divisibility.) It is this purely mathematical interpretation that underwrites the somewhat dismissive attitude characteristic of the much of the recent commentary on the Enlightenment debate over the structure of continua. But in the light of the actual parts-potential parts metaphysical background, the early modern controversy—which is accorded such central importance by major period thinkers such as Galileo, Leibniz, Hume, Euler, and Kant—is more interesting than has been allowed.

NOTES

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1 Tobias Smollett, *The History and Adventures of an Atom* (Athens: University of Georgia Press, 1989), 27.

2 David Hume, *A Treatise of Human Nature*, ed. David Fate Norton and Mary J. Norton (Oxford and New York: Oxford University Press, 2000). Hereafter abbreviated "T" with book, part, section and paragraph numbers inserted parenthetically in the text. References abbreviated "SBN" give the corresponding page numbers in the Selby-Bigge/Nidditch version: David Hume, *A Treatise of Human Nature*, ed. L. A. Selby-Bigge, 2nd ed., revised by P. H. Nidditch (Oxford: Clarendon Press, 1978).

3 John Laird, *Hume's Philosophy of Human Nature* (Archon Books, 1967), 66-7; Antony Flew, "Infinite Divisibility in Hume's *Treatise*," in *Hume: A Re-evaluation*, eds. D. W. Livingston and J. T. King (New York: Fordham University Press, 1976), 257-69; Robert Fogelin, "Hume and Berkeley on the Proofs of Infinite Divisibility," *The Philosophical Review* 97 (1988): 47-69; Marina Frasca-Spada, "Some Features of Hume's Conception of Space," *Studies in History and Philosophy of Science* 21 (1990): 371-411; Donald L. M. Baxter, "Hume on Infinite Divisibility," in *Hume: Critical Assessments*, ed. Stanley Tweyman, 6 vols. (London: Routledge, 1995), 3: 16-22.

For commentary that lays a greater emphasis on the role of Hume's other arguments against infinite divisibility, see C. D. Broad, "Hume's Doctrine of Space," Dawes Hicks Lecture on Philosophy, *Proceedings of the British Academy* 47 (1961): 171; Dale Jacquette, "Hume on Infinite Divisibility and Extensionless Individuals," *Journal of the History of Philosophy* 34 (1996): 61-78.

4 Whatever Hume's considered opinion about the existence of an external reality later in part 2, section 6, here in section 2 he is prepared to apply his reasoning to external world continua, admitting their existence at least for the sake of the argument. Compare, for instance, the title of section 1 ("*Of the infinite divisibility of our ideas of space and time*") with the title of section 2 ("*Of the infinite divisibility of space and time*"), where the lead argument occurs. And in section 4: "no idea of extension or duration consists of an infinite number of parts or inferior ideas, but of a finite number, and these simple and indivisible: 'Tis therefore possible for space and time to exist conformable to this idea: And if it be possible, 'tis certain they actually do exist conformable to it; since their infinite divisibility is utterly impossible and contradictory" (T 1.2.4.1; SBN 39). As Frasca-Spada puts it, "Hume's discussion implies some sort of presupposition about the existence of external reality. Such a presupposition is not developed, in fact it is not even enunciated, but simply underlies Hume's discussion line after line" (Marina Frasca-Spada, "Reality and the Coloured Points in Hume's *Treatise*: Part 2: Reality," *The British Journal for the History of Philosophy* 6 (1998): 25-45, 37, and *Space and the Self in Hume's Treatise* [Cambridge: Cambridge University Press, 1998], 46).

5 See, for instance, Antony Flew, "Infinite Divisibility in Hume's *Treatise*," 266. Also Frasca-Spada in the previous note.

6 Antony Flew, "Infinite Divisibility in Hume's *Treatise*," 259, 260. In a subsequent paper, Flew adds that (H1) is "nowadays . . . plainly unacceptable" ("Hume on Space and Geometry": One Reservation," *Hume Studies* 8 [1982]: 62-5, 62). Fogelin, *Hume's Skepticism in the Treatise of Human Nature* (London: Routledge and Kegan Paul, 1985), 27; see also his "Hume and Berkeley on the Proofs of Infinite Divisibility," 51. Frasca-Spada, "Some Features of Hume's Conception of Space," 397.

7 Flew, "Infinite Divisibility in Hume's *Treatise*," 260.

8 Flew, "Infinite Divisibility in Hume's *Treatise*," 259-60.

9 As we shall see shortly, the appeal to the example of the cake is quite question begging.

10 Fogelin, "Hume and Berkeley on the Proofs of Infinite Divisibility," 51.

11 T 1.2.2.2, note 6; SBN 30, note 1.

12 Laird, Flew, J. M. M. H. Thijssen, and H. Mark Pressman each join Fogelin in pressing this objection to principle (H2). See Laird, *Hume's Philosophy of Human Nature*, 65; Flew, "Infinite Divisibility in Hume's *Treatise*," 264; Thijssen, "David Hume and John Keill and the Structure of Continua," *Journal of the History of Ideas* 53 (1992): 271–86, 280; Pressman, "Hume on Geometry and Infinite Divisibility in the *Treatise*," *Hume Studies* 23 (1997): 227–44, 241.

13 James Franklin, "Achievements and Fallacies in Hume's Account of Infinite Divisibility," *Hume Studies* 20 (1994): 85–101, 87. Franklin's analysis here echoes a similar point in Bertrand Russell's discussion in *Our Knowledge of the External World* (London: George Allen and Unwin, 1926), 137, 159.

14 Peter Van Inwagen dubbed such an actual parts analysis of matter 'the doctrine of arbitrary undetached parts.' Van Inwagen rejects the existence of such parts and thus rejects the actual parts account of matter. See his "The Doctrine of Arbitrary Undetached Parts," *Pacific Philosophical Quarterly* 62 (1981): 123–37. The doctrine of arbitrary undetached parts—the actual parts account of matter—is upheld in the recent literature by Dean W. Zimmerman, Phillip Cummins, and Andrew Pyle. See Dean W. Zimmerman, "Could Extended Objects Be Made Out of Simple Parts? An Argument for 'Atomless Gunk,'" *Philosophy and Phenomenological Research* 56 (1996): 1–29 (esp. 8); Phillip Cummins, "Bayle, Leibniz, Hume and Reid on Extension, Composition and Simples," *History of Philosophy Quarterly* 7 (1990): 299–314 (esp. 304); Andrew Pyle, *Atomism and Its Critics* (Bristol: Thoemmes Press, 1995), 391. Compare also Richard Cartwright's discussion of 'the Covering Principle' and the case of fission in his "Scattered Objects," in *Analysis and Metaphysics*, ed. Keith Lehrer (D. Reidel Publishing Co., 1975), 153–71 (esp. 160–1).

15 Samuel Clarke, *A Defence of the Immateriality and Natural Immortality of the Soul*, in *The Works of Samuel Clarke*, facsimile reprint of the 1738 edition, 4 vols. (New York: Garland 1978), 3: 762.

16 Thomas Reid, *Essays on the Intellectual Powers of Man*, in *The Works of Thomas Reid*, ed. Sir William Hamilton, facsimile reprint of the 1863 edition, 2 vols. (Bristol: Thoemmes Press 1994), 1: 323.

17 "I consider the two halves of a part of matter, however small it may be, as two complete substances." Letter to Gibieuf, 19 January 1642, in *The Philosophical Writings of Descartes*, trans. John Cottingham, Robert Stoothoff, Dugald Murdoch, and Anthony Kenny, 3 vols. (Cambridge: Cambridge University Press, 1991) 3: 202–3. See also Descartes's *Principles of Philosophy*, part 1, sections 60 and 64, and part 2, section 55, in *The Philosophical Writings of Descartes*, 1: 213, 215–16, 246. Likewise the Cartesian Jacques Rohault: "When we consider a determinate Portion of Matter without Prejudice, and compare it with other Portions of Matter with which it is encompassed, we easily conceive that its particular Existence is wholly independent of those that are near it, and that it does not cease to be what it is, by being joined or united to other Portions of Matter." Jacques Rohault, *System of Natural Philosophy*, trans. John Clarke, facsimile of the 1723 edition (New

York: Johnson Reprint Co., 1969), 31. Antoine Le Grand, another great Cartesian, also endorses the doctrine: “every *Extended Being*, hath Parts distinct from each other”; “whatsoever is extended, must be conceiv’d to have distinct Parts.” Antoine Le Grand, *An Entire Body of Philosophy*, trans. Richard Blome (London: 1694), 1: 99, 97.

18 “Those things which can exist being actually separate; are really distinct; but Parts can exist being actually distinct, even before division. For Division doth not give them their peculiar Entity and Individuation, which is essential to them and the root of Distinction.” Walter Charleton, *Physiologia Epicuro-Gassendo-Charltoniana*, facsimile reprint of the 1654 edition (New York: Johnson Reprint Corporation, 1966), 109.

19 Isaac Barrow, the great seventeenth-century mathematician and Newton’s predecessor in the Lucasian Chair at Trinity, writes that “all imaginable Geometrical Figures are really inherent in every Particle of Matter; I say really inherent in Fact [*actu*] and to the utmost Perfection, though not apparent to the Sense; . . . So if the Hand of an Angel (at least the Power of God) should think fit to polish any Particle of Matter without Vacuity, a Spherical Superfice would appear to the Eyes of a Figure exactly round; not as created anew, but as unveiled and laid open from the Disguises and Covers of its circumjacent Matter.” The unveiled parts are, moreover, genuinely independent entities, for “*Divisibility* is the inseparable Companion of *Composition*. . . . Magnitude is composed of Parts really distinct; these may exist asunder, at least be considered seperately, *i.e.* they may be really or mentally dissolved or resolved into Parts.” Isaac Barrow, *The Usefulness of Mathematical Learning Explained and Demonstrated: Being Mathematical Lectures Read in the University of Cambridge*, trans. John Kirkby, facsimile reprint of the 1734 edition (London: Frank Cass, 1970), 76–7, 151; see also 148–9, 150, 162.

20 “[The nature of corporeal substance] is nothing but aliud extra aliud, ‘one thing without another,’ and therefore perfect alterity, disunity, and divisibility. So that no extensum whatsoever, of any sensible bigness, is truly and really one substance, but a multitude or heap of substances, as many as there are parts, into which it is divisible.” Ralph Cudworth, *True Intellectual System*, 3 vols. (London: Thomas Tegg, 1845), 3: 393; see also 390, 394.

21 “[E]very extension, no matter how small it may be . . . is a collection of distinct bodies.” Pierre Bayle, “Zeno of Elea” article, *Historical and Critical Dictionary*, trans. and ed. Richard H. Popkin (Indianapolis: Hackett, 1991), 360. And again: “extension is an entity that has parts distinct from each other”; “*extension is composed of parts that are each a particular substance*” (“Leucippus” article, in the *Dictionary*, 138, “Spinoza” article, in the *Dictionary*, 305). See also the *Dictionary*, 302, 338, 304, 306, and the *Système Abrégé*, in *Oeuvres Diverses de Pierre Bayle*, facsimile reprint of the 1731 edition, 4 vols. (Hildesheim: Georg Olms, 1968), 4: 297, 299.

22 The doctrine is fundamental to Leibniz’s system: one could provide dozens of quotes; I will give just two. “Everyone agrees that matter has parts, and consequently that it is a *multitude* of many substances, as would be a herd of sheep.” Letter to Electress Sophie, in *Die philosophischen Schriften von Gottfried Wilhelm*

Leibniz, ed. C. I. Gerhardt, 7 vols. (Berlin: Weidman 1875–90), 7: 552; quoted and translated by Donald Rutherford in his *Leibniz and the Rational Order of Nature* (Cambridge: Cambridge University Press, 1995), 220. “A body is not a substance but an aggregate of substances, since it is always further divisible, and any given part always has another part, to infinity.” G. W. Leibniz, *Notes on Some Comments by Michel Angelo Fardella*, in *Philosophical Essays*, trans. Roger Ariew and Daniel Garber (Indianapolis: Hackett 1989), 103.

23 In an early work Newton deploys an argument that implicitly assumes the actual parts analysis of matter: see his *Certain Philosophical Questions: Newton's Trinity Notebook*, trans. and ed. J. E. McGuire and Martin Tamny (Cambridge, England: Cambridge University Press, 1983), 341. We have already seen an endorsement of the doctrine from Samuel Clarke in the main body of the text. To this we could add the following quotation from Clarke's correspondence with Leibniz (famously written under Newton's guidance): “Parts, in the corporeal sense of the word, are separable, compounded, ununited, independent of and moveable from, each other.” *The Clarke-Leibniz Correspondence*, ed. H. G. Alexander (Manchester: Manchester University Press, 1956), 48. Similarly the theologian and card-carrying Newtonian William Wollaston writes that “*Matter exists in parts, . . .* These *parts*, tho they are many times kept closely united by some occult influence, are in truth so many *distinct* bodies, which may, at least in our imagination, be disjoined or placed otherwise: nor can we have any idea of matter, which does not imply a natural discernibility.” William Wollaston, *The Religion of Nature Delineated*, facsimile reprint of the 1724 edition (Delmar, N.Y.: Scholars' Facsimiles and Reprints, 1974), 74. And the great Newtonian and mathematician Leonhard Euler: “[Wolffian monadists] denominate [bodies] compound beings, which no one can deny, as extension necessarily supposes divisibility, and consequently a combination of parts which constitute bodies.” Leonhard Euler, *Letters of Euler on Different Subjects in Natural Philosophy* (New York: Arno Press, 1975), 59.

24 See Bayle, *Système Abrégé*, in *Oeuvres Diverses de Pierre Bayle*, 4: 299 for statements of all four of these arguments. See also Charleton, *Physiologia Epicuro-Gassendo-Charletoniana*, 93, 108 for versions of the first and second arguments.

25 For a recent defense of the potential parts account of matter, see William Charlton, “Aristotle's Potential Infinities,” in *Aristotle's Physics: A Collection of Essays*, ed. Lindsay Johnson (Oxford: Oxford University Press, 1991), 133–4. Compare also Peter Van Inwagen in note 14 above.

26 Simplicio, the orthodox Aristotelian character in Galileo's *Dialogues Concerning Two New Sciences*, presents the potential parts analysis of matter as follows: “Parts cannot be said to be in a body which is not yet divided or at least marked out; if this is not done we say that they exist potentially.” Galileo Galilei, *Dialogues Concerning Two New Sciences*, trans. Henry Crew and Alfonso de Dalvio (New York: Macmillan, 1914), 34.

Sir Kenelm Digby (no less a party-line Aristotelian than Simplicio on this particular issue) also gives us a clear statement of the view. Any given body, he writes, “is but one *whole* that may indeed be cutt into so many severall partes: but those partes are not really there, till by division they are parcelled out: and then, the

whole (out of which they are made) ceaseth to be any longer; and the partes succede in lieu of it; and are, every one of them, a new *whole*." And again, "the partes which are considered in Quantity, are not diverse thinges: but are onely a vertue or power to be diverse thinges." Kenelm Digby, *Two Treatises, In the one of which the Nature of Bodies is expounded; in the other, the Nature of Man's Soule; is looked into*, facsimile of the 1644 edition (Stuttgart: Freidrich Fromman Verlag, 1970), 10, 13. Digby—along with his mentor Thomas White—develops a two-sided system of natural philosophy that, in part, employs the corpuscularian-mechanistic methods of the new science and, in part, invokes Aristotelian-scholastic models of explanation. On the present issue—the structure of physical continua—the *Two Treatises* is strictly Aristotelian.

Our final statement of the view is from Hobbes: "it is manifest, that nothing has parts till it be divided; and when a thing is divided, the parts are only so many as the division makes them." Hobbes, *De Corpore* section 7.9, in *The Collected Works of Thomas Hobbes*, ed. Sir William Molesworth, 11 vols. (London: John Barth, 1839–41), 1: 97. See also *Leviathan*, in *Works*, 3: 677, where division again 'makes' parts.

27 See, for instance, Digby, *Two Treatises*, 9–13, for statements of all three arguments. For more on the Aristotelian-scholastic version of the potential parts doctrine, see Theodore Scaltsas, *Substance and Universals in Aristotle's Metaphysics* (Ithaca, N.Y.: Cornell University Press, 1994), 83–7.

28 I have subsequently found that Donald L. M. Baxter makes a similar point in his paper "Hume on the Simplicity of Moments" (unpublished draft). Thanks to Baxter for showing me this draft.

29 Hume seems to have simply taken it for granted that ultimate parts must all be the same size, and—as far as I can tell—the same is true of those other early moderns who also endorse the overall actual parts argument against infinite divisibility (see the references in notes 32 and 33 below). This assumption may well seem a natural one, particularly if we are talking about ultimate noncomposite parts that are somehow true simples and logically or conceptually indivisible, not merely physically unsplitable. Perhaps these philosophers held (implicitly) that nothing can be such an ultimate simple part if something smaller than it exists, and hence that all ultimate parts must be the same size. In any case, this assumption of the uniformity of ultimate parts is clearly crucial to Hume's overall case against infinite divisibility, and constitutes a vulnerable stage of the argument should anyone be prepared to countenance ultimate parts of different sizes and hence the possibility of a series of ultimate simples that fall off in size proportionately toward a limit (as in the series $1/2, 1/4, 1/8 \dots$). So here we can say that the proportional parts objection will come back to haunt Hume unless he can supply a reason why the noncomposite first parts must be uniform in size.

30. Donald L. M. Baxter has stressed the crucial role of this argument for ultimate parts in Hume's overall case against infinite divisibility, and has also noted that it presupposes the principle that "anything divisible is composed of parts": "that *divisible* entails *not unitary*" (i.e., the actual parts doctrine). See his "Hume on Infinite Divisibility," in Tweyman, *Hume: Critical Assessments*, 3: 19.

31 Notice that Kant stresses that this argument for an atomic base of simples only goes through if one thinks of the extended whole as a “*compositum*” rather than as a “*totum*”—that is to say, if one thinks of it having actual rather than potential parts. Immanuel Kant, *Critique of Pure Reason*, trans. Werner S. Pluhar (Indianapolis: Hackett, 1996), B466–8.

32 Henry More, *Antidote against Atheism*, in *Philosophical Writings of Henry More*, ed. F. I. Mackinnon (New York: Oxford University Press, 1925), 11. Joseph Glanvill, *The Vanity of Dogmatizing*, in *The Collected Works of Joseph Glanvill* (Hildesheim: Georg Olms Verlag, 1970), 53. Newton, *Certain Philosophical Questions*, 341. Bayle, *Historical and Critical Dictionary*, 362. George Berkeley, *Principles*, section 47 and *Philosophical Commentaries*, entry 352, in *The Works of George Berkeley*, ed. A. A. Luce and T. E. Jessop, 9 vols. (London: Thomas Nelson and Sons, 1948–57), 2: 60, and 1: 42. Isaac Barrow, *The Usefulness of Mathematical Learning Explained and Demonstrated: Being Mathematical Lectures Read in the University of Cambridge*, trans. John Kirkby, facsimile reprint of the 1734 edition (London: Frank Cass, 1970), 157. John Keill, *An Introduction to Natural Philosophy* (London: Senex, Innys, Manby, Osborn, and Longman, 1733), 35–7. The argument is at least as old as Epicurus: see his *Letter to Herodotus*, paragraph 57, in *The Epicurus Reader*, trans. and ed. Brad Inwood and L. P. Gerson (Indianapolis: Hackett, 1994).

33 Digby, *Two Treatises*, 10–12. Charleton, *Physiologia*, 91–3. Bayle, *Historical and Critical Dictionary*, 362, 356. Charleton and Bayle endorse the actual parts doctrine and hence this argument against infinite divisibility. For Digby, the conclusion that bodies cannot be infinitely divisible constitutes a *reductio* on the actual parts doctrine.

34. In Charleton’s text, the abbreviated version runs as follows: “If in a Finite Body, the number of Parts, into which it may be divided, be not Finite also; then must the Parts comprehended therein be really Infinite: and, upon Consequence, the whole Composition resulting from their Commixture, be really Infinite; which is repugnant to the supposition.” Charleton, *Physiologia*, 91.

35 Charleton, *Physiologia*, 92, 93.

36 Charleton, *Physiologia*, 94, 91.

37 On Hume’s familiarity with Barrow’s *Lectures* and Bayle’s “Zeno” article, see Norman Kemp Smith, *The Philosophy of David Hume* (London: Macmillan, 1941), 325–6, 343. On Keill’s fame and influence and at least a “very tenuous” link to Hume, see Thijssen, “David Hume and John Keill and the Structure of Continua,” 272–3.

38 Barrow, *The Usefulness of Mathematical Learning Explained and Demonstrated*, 157; Bayle, *Historical and Critical Dictionary*, 356; Keill, *An Introduction to Natural Philosophy*, 37, 40–41.

39. T 1.2.4.3; SBN 40.

40 T 1.1.7.3; SBN 18. My emphasis.

41 R. Newman, "Hume on Space and Geometry," in *Hume: Critical Assessments*, ed. Tweyman, 3: 43. Compare also Wayne Waxman, "The Psychologistic Foundations of Hume's Critique of Mathematical Philosophy," *Hume Studies* 22 (1996): 123–67 (esp. 142–9).

42 R. Newman, "Hume on Space and Geometry," 44 (my italics).

43 See note 4. It is also worth noting that in the two places where Hume *does* explicitly set out the argument from phenomenological atomism against the infinite divisibility of ideas and impressions, the context is quite different from the discussion of the lead argument (H1)–(H3). The argument from phenomenological atomism occurs first at T 1.2.1.2–3 (SBN 26–7), where it expressly concerns "the infinite divisibility of *our ideas* of space and time" (T 1.2.1 section title; SBN 26; my emphasis). The lead argument, by contrast, concerns "the infinite divisibility of space and time" (T 1.2.2 section title; SBN 29). The argument from phenomenological atomism then appears again at T 1.2.3.12–15 (SBN 38), where it is introduced—in apparent contrast to the lead argument—as "*another* very decisive argument," and again expressly concerns the divisibility of "*our ideas* of space and time" (T 1.2.3.12; SBN 38; my emphasis).

44 R. Newman, "Hume on Space and Geometry," 42.