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On the Compatibility between Euclidean Geometry and Hume's Denial of Infinite Divisibility

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Abstract: It has been argued that Hume's denial of infinite divisibility entails the falsity of most of the familiar theorems of Euclidean geometry, including the Pythagorean theorem and the bisection theorem. I argue that Hume's thesis that there are indivisibles is not incompatible with the Pythagorean theorem and other central theorems of Euclidean geometry, but only with those theorems that deal with matters of minuteness. The key to understanding Hume's view of geometry is the distinction he draws between a precise and an imprecise standard of equality in extension. Hume's project is different from the attempt made by Berkeley in some of his later writings to save Euclidean geometry. Unlike Berkeley, who interprets the theorems of Euclidean geometry as false albeit useful approximations of geometrical facts, Hume is able to save most of the central theorems as true.

In the *Treatise*, David Hume denies the thesis that extension is infinitely divisible, even though it can be derived as a theorem of Euclidean geometry. This clearly shows that he rejects some of the theorems of Euclidean geometry. What is less clear is the extent to which he thinks geometry needs to be revised. It has been argued that Hume's rejection of infinite divisibility entails that most of the familiar theorems of Euclidean geometry, including the Pythagorean theorem and

the bisection theorem, are false, a view that is normally associated with Berkeley's earlier writings.

I argue that Hume's denial of infinite divisibility is not incompatible with the Pythagorean theorem and other central theorems of Euclidean geometry, but only with those theorems that deal with matters of minuteness. Hume's project is much less radical than the discrete geometry project attempted by Berkeley in some of his earlier writings.¹ In fact, Hume rejects the discrete geometry project as impossible. Moreover, his account of geometry is different from Berkeley's attempt in his later writings to save the theorems of Euclidean geometry as useful approximations of the relations that hold between geometrical figures. For Hume, the Pythagorean theorem, as well as all other theorems except those that go beyond a certain level of precision, is true, not merely approximately true.

Tensions between Euclidean Geometry and the Indivisibles

Infinite divisibility is the thesis that line segments are divisible ad infinitum and is equivalent, for Hume, to the thesis that there are no indivisible entities. Although these contentious entities are more frequently called mathematical points or minima sensibilia, I will refer to them as indivisibles.² On the one hand, Hume was certainly aware that there is a tension between Euclidean geometry and the denial of infinite divisibility, because the infinite divisibility thesis itself is a theorem of Euclidean geometry. There are many alternative proofs of infinite divisibility that Hume was familiar with, for instance, the proof derived from the incommensurability between the diagonal and the sides of a square. If there were indivisibles, then the diagonal would be commensurable with the sides.³ On the other hand, there are passages in the *Treatise* which suggest that Hume remains committed to most of the familiar theorems of Euclidean geometry:⁴

The three angles of a triangle are equal to each other . . . [is] true with relation to that idea [of a particular equilateral triangle], which we had form'd. (T 1.1.7.8; SBN 21)

'Tis from the idea of a triangle, that we discover the relation of equality, which its three angles bear to two right ones; and this relation is invariable, as long as our idea remains the same. (T 1.3.1.1; SBN 69)⁵

[G]eometry fails of evidence in this single point, while all its other reasonings command our fullest assent and approbation. (T 1.2.4.31; SBN 52)

Nevertheless, it has been held that there is a deeper conflict between Euclidean geometry and the rejection of infinite divisibility, characterized by what I call the Deep Incompatibility (DI) thesis:

(DI) The denial of infinite divisibility requires a radical revision of Euclidean geometry.

It would be a difficult task to clarify what it means for a revision of Euclidean geometry to be radical. In any case, the rejection of central theorems such as the Pythagorean theorem would count as a radical revision of geometry. Since Hume undoubtedly rejects infinite divisibility, DI would require him to carry out a more radical revision of geometry than he is willing to admit. At the time he wrote the *Philosophical Commentaries*, Berkeley took DI seriously and thought that his commitment to indivisibles required him to reject Euclidean geometry and replace it by discrete geometry. It is not hard to see how DI can be argued for. Commenting on Berkeley, Jesseph⁶ writes:

If we accept the doctrine of the minimum sensible, most geometric theorems will be literally false, not only of particular lines and figures employed in a proof but, because no irrational proportion between any two finite collections of minima can be established, of any perceivable or imaginable lines or figures which they might represent. (Jesseph, 77)

In particular, it has been argued that the commitment to indivisibles requires one to drop familiar theorems such as the bisection theorem and the Pythagorean theorem. Euclidean geometry has it that any line segment can be bisected (proposition I.10 in Euclid's *Elements*). One standard way to prove in Euclidean geometry that an arbitrary line segment, AB, can be bisected is to construct two circles centered on A and B and having radius AB, and unite the points where the two circles intersect (see fig. 1).⁷

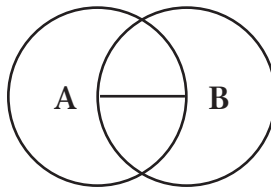


Figure 1

Jacquette⁸ argues that Berkeley and Hume would have to reject both the bisection and the bipartition⁹ of arbitrary line segments for the following reason. Consider an arbitrary line segment. If it is finitely divisible, then it contains either an even or an odd number of indivisibles. If it contains an even number of indivisibles, then it can be bipartitioned, but not bisected, since it has no midpoint at which to be bisected. If it contains an odd number of indivisibles, then it can be bi-

sected, but cannot be bipartitioned, since it cannot be divided into two equal subsegments. Jacquette believes that this and other similar reasons show that Hume's commitment to indivisibles leads him to deny most of the theorems of Euclidean geometry:

By denying this [bisection and bipartition], Hume strays from classical continuous geometry in the direction of a Berkeleyan discrete geometry, which he nowhere acknowledges or tries to develop. . . . From the standpoint of Hume's critique of infinity, the theorems of classical continuous geometry are only approximately true. (Jacquette, 178–79)

Similarly, Pressman¹⁰ argues that there is a conflict between Hume's thesis that there are indivisibles and the Pythagorean theorem:

[E]ither the Pythagorean theorem . . . fails, or Hume's thesis that segments contain finitely many points fails. (Pressman, 239)

The reason is, allegedly, that a right triangle with sides consisting of 100 indivisibles should have, according to the Pythagorean theorem, a hypotenuse consisting of 141.42135 . . . indivisibles. However, no line segment can consist of 141.42135 . . . indivisibles, because this would mean that the indivisibles have parts.

The case for a deep incompatibility between indivisibles and Euclidean geometry appears to be very strong. It is true that, in his later writings, Berkeley develops a theory of representative generality which allows him to save the usefulness of Euclidean geometry while remaining committed to the finite divisibility of extension. However, this theory fails to refute the deep incompatibility view. According to the theory of representative generality developed in the *Principles* and in the *Analyst*, the immediately perceived lines do not play the role of objects of geometry, but that of representations of other lines that are not actually perceived. This provides Berkeley with a way to avoid the difficulties related to the number of indivisibles in an actually perceived figure. To use Jesseph's words, "because the square we construct in a proof of the theorem is supposed to represent all possible squares, we need not be concerned with the number of minima contained in the side and diagonal of any particular one" (Jesseph, 74). Although "for any given square, there will be a rational proportion between the number of minima along the diagonal and the number along the side," the thesis of the incommensurability between the diagonal of a square and its side remains useful, because "as we consider larger and larger squares, the proportion will approach a limit, namely $\sqrt{2}$ " (Jesseph, 74).

The most plausible interpretation of Berkeley's view is that the theorems of geometry are, strictly speaking, false but useful on instrumentalist grounds (as approximations of the relations between geometrical figures). It is also possible to

interpret the theorems of Euclidean geometry as true statements of the limits that can never be reached, but which are increasingly closer approximated by the relations between figures with the increase in their size. Although interpreted in this way the theorems can be said to be true, this constitutes a radical departure from the traditional way of understanding the theorems of geometry as statements of relations that hold between particular geometrical figures. In either case, Berkeley's theory of representative generality fails to refute the thesis of deep incompatibility between finite divisibility and Euclidean geometry.

The Compatibility between Finite Divisibility and the Pythagorean Theorem

Hume believes that the deep incompatibility thesis fails and that he is not forced to carry out a radical revision of Euclidean geometry. How is it possible to resist the arguments for deep incompatibility reviewed in the previous section? One answer to Pressmann's objection that the rejection of infinite divisibility is incompatible with the Pythagorean theorem has been put forward by Falkenstein.¹¹ He argues that the Pythagorean theorem is, in fact, compatible with the denial of infinite divisibility. This is possible if one drops the assumption that for any two natural numbers, n and m , there is a right triangle whose two sides on the left and the right of the right angle consist of n and m indivisibles, respectively. There are right triangles with sides consisting of three and four indivisibles (the hypotenuse consists in this case of five indivisibles), but there is no right triangle with two sides each consisting of 100 indivisibles, because there can be no side consisting of 141.42135 indivisibles. Although this strategy can be used to save the Pythagorean theorem, it cannot be used to save the bisection theorems or many other central theorems of Euclidean geometry, so it cannot be used to refute the deep inconsistency thesis. Moreover, there are good reasons to think that this is not the reason why Hume believes that the Pythagorean theorem is true.¹² As I will argue, he explicitly denies the possibility of a discrete geometry of the sort proposed by Berkeley in his earlier work.

Hume accounts for the compatibility between most of the familiar theorems of Euclidean geometry and finite divisibility in an original way. His crucial idea is to distinguish between two standards of equality in extension: a precise and an imprecise standard. The precise standard of equality takes two lines to be equal if and only if they consist of the same number of indivisibles. It is this standard of equality that leads one to the rejection of most of the theorems of Euclidean geometry. In particular, it led Berkeley to the discrete geometry project advocated in the *Philosophical Commentaries*. Hume explicitly denies that this standard of equality can be properly used in geometry on the grounds that the mind is not able to count the smallest parts of a line segment:

[Y]et I may affirm, that this standard of equality is entirely *useless*, and that it never is from such a comparison we determine objects to be equal or unequal with respect to each other. For as the points, which enter into the composition of any line or surface, whether perceiv'd by the sight or touch, are so minute and so confounded with each other, that 'tis utterly impossible for the mind to compute their number, such a computation will never afford us a standard, by which we may judge of proportions. (T 1.2.4.19; SBN 45)

As Jacquette correctly notes, “Hume does not propose a nonclassical discrete geometry such as Berkeley suggests in the notebooks of his *Philosophical Commentaries*” (Jacquette, 176).

The other standard of equality is more similar to the standard of equality used in Euclidean geometry, but differs from it in that it is imprecise. In Euclidean geometry equality was typically thought of as a precise notion which can be cashed out in terms of congruity or superposition: two line segments are equal if and only if they can be superposed upon one another.¹³ Whether this is supposed to provide an explicit definition or merely a standard that can be used to test equality is controversial. Euclid himself seems to have thought of equality as being only implicitly defined by a set of five axioms, the fourth of which is the superposition criterion.¹⁴ What matters here is that superposition was taken seriously by geometers whose work Hume was very familiar with, for instance, Isaac Barrow who thinks of congruity as “the chief Pillar and principal Bulwark of all the Mathematics.”¹⁵ Hume, however, denies that geometers could use a precise standard of equality derived from congruity. If congruity presupposes that “two figures are equal, when upon the placing of one upon the other, all their parts correspond to and touch each other” (T 1.2.4.21; SBN 46), then the standard of equality would indeed be precise, but it would coincide with the standard based on the number of indivisibles, which he already rejected as useless. Hume is best understood as holding that the standard of equality which is proper to geometry is imprecise, because the comparison of two line segments relies in an essential way on mere appearances:

[T]he very idea of equality is that of such a particular appearance corrected by juxta-position or a common measure. (T 1.2.4.24; SBN 48)

Our appeal is still to the weak and fallible judgment, which we make from the appearance of the objects, and correct by a compass or common measure. (T 1.2.4.29; SBN 51)

According to Hume, judgments of equality based on appearances “are not only common, but in many cases certain and infallible” (T 1.2.4.22; SBN 47). Very

often the judgments based on appearances can be corrected “by a review and reflection,” “by juxta-position,” or “by the use of some common and invariable measure” (T 1.2.4.23; SBN 47), but we will never be able to go beyond a certain level of precision. As Hume points out, this way of understanding the standard of equality “renders our imagination and senses the ultimate judges of it” (T Abs.29; SBN 659), which sits well with the idea that geometry is an empirical science. Just like Euclidean geometry, Hume’s geometry aims at providing general truths describing the properties of all possible geometrical objects. The difference is that in Hume’s geometry generality is not derived from the abstract nature of the objects but rather is achieved by abstracting from the geometrically irrelevant properties of particular appearances.

It is important to notice that, for Hume, there are other central notions of geometry (the idea of a right line and that of a plane surface) which are derived from an imprecise standard:¹⁶

[N]either is this [the precise standard based on indivisibles] the standard from which we form the idea of a right line; . . . The original standard of a right line is in reality nothing but a certain general appearance. (T 1.2.4.30; SBN 52)

It appears, then, that the ideas which are most essential to geometry, *viz.* those of equality and inequality, of a right line and a plain surface, are far from being exact and determinate, according to our common method of conceiving them. (T 1.2.4.29; SBN 50–51)

This means that there is no sharp boundary between the perfectly straight lines and the very slightly curved ones. Moreover, although as particular appearances right lines (just like any other impressions of the senses and ideas copied from them) are “clear and precise” (T 1.3.1.7; SBN 72), they are not precise with respect to the number of indivisibles. When he points out that the mind cannot compute the number of indivisibles in a line segment, Hume is already committed to the idea that, as Kemp Smith puts it, “the addition or removal of one of the minima sensibilia is not discernible either in appearance or in measurement” (303). The fact that although a right line consists of indivisibles distributed in a certain order, “nothing is observ’d but the united appearance” (T 1.2.4.25; SBN 49) shows that for him this is not a merely epistemological barrier but a metaphysical one: geometry does not deal with sharply defined notions such as a line segment consisting of 100 indivisibles.

The distinction between the two standards enables Hume to reject some of the theorems of Euclidean geometry and, at the same time, save most of the familiar theorems as true. For him, a proof cannot guarantee the truth of its conclusion if the

degree of precision that has been used to interpret the conclusion exceeds the degree of precision that corresponds to the premises. Since the postulates and objects of geometry are imprecise, a geometrical proof is fallacious when its conclusion goes beyond a certain level of intricacy or beyond a certain level of minuteness:

[W]ith regard to such minute objects, they [the proofs] are not properly demonstrations, being built on ideas, which are not exact,¹⁷ and maxims, which are not precisely true. (T 1.2.4.17; SBN 45)

In particular, geometrical proofs¹⁸ of infinite divisibility are fallacious because their conclusion presupposes that certain distinctions can be made even at the level of extremely small entities and, for this reason, requires to be interpreted in accordance with the precise standard. In other cases, though, geometry remains a source of certainty and demonstrative knowledge.¹⁹

Fogelin offers a similar interpretation of Hume when he argues that his geometry is an empirical discipline which “for the most part yields proofs that are quite beyond doubt . . . [but] it can lead to error when its reflections are carried beyond the observational claims that serve as its basis” (Fogelin, 56–57). Nevertheless, he does not go so far as distinguishing between the two standards of equality (although this would be the natural way to develop the view). The lack of precision is, for Fogelin, due to the fact that in the *Treatise* geometrical reasoning is not purely demonstrative but relies on observation.²⁰ He also claims that in the *Enquiry* Hume changed his mind and conceived of geometry as a purely demonstrative discipline which is “derived wholly from Relations of Ideas” (Fogelin, 57). I think that the imprecision of geometry has, for Hume, a deeper source than its reliance on the observation of physical entities. It rather stems from the fact that the standard of equality as well as that of a right line or of a plane surface is imprecise. Even if geometry is confined to relations of ideas, it remains an inexact science.²¹

I will now turn to the other theorems which have been held to be in conflict with Hume’s geometry. Unlike the infinite divisibility thesis, the Pythagorean theorem does not explicitly deal with extremely small entities, but it can be read as involving either a precise or an imprecise notion of equality. If the equality involved in the theorem is understood in accordance with a precise standard of equality,²² then all proofs formulated in Euclidean geometry fail to establish that the theorem holds, for the same reason that the proof of infinite divisibility fails to establish its truth. On the other hand, if the equality involved in the Pythagorean theorem is understood according to the imprecise standard, then some proofs of the theorem²³ constitute a genuine demonstration of a truth of geometry. Therefore, if it is not taken as a statement of precise equality, the Pythagorean theorem holds. Hume does not have to adopt the strategy suggested by Falkenstein and claim that some given line segments cannot form two sides of a right triangle. In

fact, to argue that the Pythagorean theorem can be used to show that there is no right triangle each of whose two sides consist of 100 indivisibles is to commit the same mistake made by those geometers who are persuaded by geometrical proofs that extension is infinitely divisible. Geometry would be taken to yield results that go much beyond its standards of precision.

The fact that geometry has an imprecise standard of equality in extension does not mean that its propositions are mere approximations of the states of affairs. This view would be more akin to Berkeley's theory of representative generality and would entail that the theorems of geometry are most of the time false. Nothing in what Hume says suggests that he endorses or is influenced by this theory. Instead, Hume should be interpreted as saying that the demonstrations of geometry provide true statements of imprecise equality. In particular, one should not think of the Pythagorean theorem as making a false statement of precise equality which approximates the truth, but as the true statement that some quantities are roughly equal (i.e., indiscernible by the imprecise standard).²⁴ It might be thought that this is merely a linguistic difference on the grounds that talk about false statements which approximate the truth can always be converted into talk about true statements of approximate equality and the other way around.²⁵ For instance, saying that the statement that two persons, A and B, are the same height is false but approximates the truth might appear to be equivalent to the true statement that the heights of A and B are roughly equal. Nevertheless, the distinction is significant and helps clarify Hume's view about the objects and the status of geometry. First of all, if ' $=_p$ ' and ' $=_i$ ' stand, respectively, for precise and imprecise equality in extension, and a and b are two roughly equal line segments, ' $a =_i b$ ' is true while ' $a =_p b$ ' is not. Hume wants geometry to provide true theorems. Second, moving to the metalinguistic level, the statement that ' $a =_p b$ ' is false but approximates the truth entails the statement that ' $a =_i b$ ' is true, but not the other way around. The reason for this asymmetry is that the former statement entails that there is a fact of the matter as to whether a and b are precisely equal, while the latter does not. There is a fact of the matter only if the objects that are compared with one another are sharply defined with respect to the number of indivisibles, but this is not the case with the line segments of Hume's geometry.

In a similar way, it is possible to show that Jacquette's argument that not every line segment can be both bisected and bipartitioned does not apply to Hume because it assumes a precise standard of equality. Since Hume's geometry is based on an imprecise standard, the distinction between line segments consisting of an even as opposed to an odd number of indivisibles cannot be made. Although this observation suffices to save Hume's geometry from Jacquette's argument, the bisection theorem needs to be treated with more care than the Pythagorean theorem, because it is sometimes identified with the infinite divisibility thesis, which Hume explicitly rejects. Jesseph, for instance, writes: "we state the thesis of infinite

divisibility, not as the claim that every magnitude contains an infinite number of parts, but as the assertion that any magnitude can be bisected” (Jesseph, 51). Nevertheless, the two theses should be and in fact have been distinguished by other commentators. Proclus,²⁶ for instance, distinguishes between the bisection theorem, the thesis that every continuous magnitude is divisible and the infinite divisibility thesis. The thesis that every continuous magnitude is divisible is, for him, an axiom and is relied on in the proof of the bisection theorem. The infinite divisibility thesis, on the other hand, is a theorem which does not play any role in that proof. Moreover, Proclus’s proof of the infinite divisibility thesis relies on the incommensurability of the side and diagonal of a square rather than on the bisection theorem. However, even if the bisection theorem and the infinite divisibility thesis are carefully distinguished from one another, the worry remains that the latter is a consequence of the former. After all, it seems that if extension were only finitely divisible, then the bisection process would have to reach its limits after a finite number of steps. I think that Hume does have enough resources to save the bisection theorem under an imprecise reading (which restricts it such that it does not apply to extremely small line segments) without being committed by this to infinite divisibility. Euclidean geometry assumes that the procedure which has been used to prove the bisection theorem (fig. 1) can be repeated *ad infinitum* beyond any degree of minuteness. This is the assumption that would be rejected by Hume. Unlike the Pythagorean theorem and the bisection theorem (which are open to both a precise and an imprecise reading), the infinite divisibility thesis allows only one reading. Given that the thesis is explicitly concerned with indivisibles, it cries out for a precise standard of equality.

The appeal to the imprecise standard enables Hume to save most of the familiar theorems of Euclidean geometry and thus refute the deep incompatibility thesis. He rejects only those theorems that involve matters of minuteness and therefore presuppose a precise standard of equality. Not all theorems of geometry deal explicitly with equality though. For instance, the axiom according to which any two points can be joined by a unique straight line (which is postulate I.1 in the *Elements*) does not make explicit reference to equality.²⁷ The axiom can be read as making an indirect equality claim²⁸ (if *a* and *b* are straight lines each of which contains two points, *A* and *B*, then the angle between them equals zero), but it is more natural to interpret it as a statement of identity (if *a* and *b* are straight lines which join two given points, then *a* is identical to *b*). Interpreted as a statement of identity, the axiom is, for Hume, false. He claims that since the standard of a right line is derived from mere appearances, it is possible for two lines approaching at the rate of an inch in twenty leagues to both meet the standard and have a segment in common.²⁹ Hume presumably operates with a precise standard of identity according to which *a* and *b* are identical only if they consist of exactly the same indivisibles. Nevertheless, his geometry is able to preserve the truth of

the axiom if it is read not as a statement of precise identity but rather as the statement that the differences between a and b “can never be of any consequence” (T 1.3.1.6; SBN 72). The differences between the two lines approaching at the rate of an inch in twenty leagues are negligible for most practical purposes and for this reason they can be treated as roughly identical. Likewise, although two points, A and B, can be united by more line segments which meet the imprecise criterion of a straight line, the differences among them are so small that they can be safely ignored. Other axioms or theorems can be handled in a similar way.

As a final remark, I should stress that by arguing that the deep incompatibility objection can be satisfactorily answered, I am not committed to claiming that Hume's view of geometry is correct or that it does not raise other problems.³⁰ In his later years, Hume himself grew dissatisfied with the *Treatise* and, if his confession is sincere, was persuaded by Lord Stanhope that there were some problems with the way he conceived of geometry.³¹ What I am committed to is that the *Treatise* contains an original empiricist view of geometry, different from Berkeley's, which allows no immediate refutation and deserves more attention than it has received in the past.

NOTES

Special thanks are due to John Biro for many stimulating discussions and for comments on an earlier draft of this paper. I also benefited from very helpful comments from Samuel Levey and from the editors and the referees of this journal.

1 See George Berkeley, “Philosophical Commentaries (1707–1708),” in *George Berkeley's Philosophical Works*, ed. M. R. Ayers (London: Everyman's Library, 1975).

2 A first reason is that Hume uses “mathematical points” in two different ways: sometimes it is used to refer to indivisibles (which are parts of extension endowed with properties such as color or solidity), while some other times it stands for the abstract extensionless entities devoid of color and sensibility, whose existence he denies. His rejection of mathematical points understood in this latter sense is important because their existence would be compatible with infinite divisibility (current geometry is, in fact, committed to both infinite divisibility and the existence of abstract extensionless points). This is not the case with Hume's indivisibles, because the idea of extension arises from the repetition of these smallest entities and an extension consisting of an infinite number of indivisibles would be infinite. See, for instance, T 1.1.2.2 (SBN 29), especially footnote 1. The second reason is that “minima sensibilia” is too restrictive, since indivisibles could be either ideas of the imagination (*minima imaginabilia*) or impressions of the senses (*minima sensibilia*). References to the *Treatise* and the *Enquiry* (abbreviated as “T” and “EHU,” respectively) are to the Book, part, section, and paragraph (or, for the *Enquiry*, only the section and the paragraph) of the Clarendon Edition of David Hume's two works: *A Treatise of Human Nature*, ed. David Fate Norton

and Mary J. Norton (Oxford: Oxford University Press, 2007), and *An Enquiry Concerning Human Understanding*, ed. Tom L. Beauchamp (Oxford: Oxford University Press, 2000). They also include the page numbers of the Selby-Bigge editions (abbreviated “SBN”): *A Treatise of Human Nature*, ed. L. A. Selby-Bigge, 2nd ed., revised by P. H. Nidditch (Oxford: The Clarendon Press, 1978), and *Enquiries Concerning Human Understanding and Concerning the Principles of Morals*, ed. L. A. Selby-Bigge, 3rd ed., revised by P. H. Nidditch (Oxford: Oxford University Press, 1975).

3 Robert Fogelin’s “Hume and Berkeley on the Proofs of Infinite Divisibility,” *The Philosophical Review* 97.1 (1988): 47–69, includes a useful discussion of some of the mathematical proofs of infinite divisibility. The proof derived from incommensurability is extracted from *The Port-Royal Logic*, but can also be found in the excerpts from Malezieu’s *Elements de Geometrie* reprinted in Kemp Smith, *The Philosophy of David Hume* (London: Macmillan, 1941), 340–42. Fogelin, “Hume and Berkeley,” 49, also mentions a proof from concentric circles found in Isaac Barrow’s *Lectures*. The idea is, roughly, that if there were indivisibles, concentric circles would consist of the same number of indivisibles, as can be shown by drawing radii from the center to each indivisible on the largest circle.

4 In the *Enquiry*, Hume’s commitment to the truth of the familiar theorems of Euclidean geometry is more explicit: “That the square of the hypotenuse is equal to the square of the two sides is a proposition which expresses a relation between these figures. . . . [T]he truths demonstrated by Euclid would for ever retain certainty and evidence” (EHU 4.1; SBN 108). Various commentators have held that there is a significant difference between the views of geometry developed by Hume in the *Treatise* and in the *Enquiry*. They argue that in the *Treatise* geometry is treated as an imprecise science which fails to provide certainty, while in the *Enquiry* its status as an exact and certain science is restored. See Anthony Flew, *Hume’s Philosophy of Belief* (New York: The Humanities Press, 1961), 62–66; Rosemary Newman, “Hume on Space and Geometry,” in *David Hume. Critical Assessments*, vol. 3, ed. S. Tweyman (London: Routledge, 1995), 3–61; and Fogelin, 47–69. In another paper, currently under review, I argue that there is no significant difference between the *Treatise* and the *Enquiry* and that both take geometry to be a source of certain albeit not exact knowledge. Here I am concerned only with Hume’s view in the *Treatise*. Hume’s denial of the infinite divisibility thesis is explicit in both the *Treatise* and the *Enquiry*.

5 This theorem is proposition I.32 in Euclid’s *Elements*. See *The Thirteen Books of Euclid’s Elements* (New York: Dover Publication: 1956), 316.

6 Douglas Jesseph, *Berkeley’s Philosophy of Mathematics* (Chicago: University of Chicago Press, 1993).

7 This proof is attributed to Apollonius and is different from Euclid’s. Both proofs can be found in *The Thirteen Books of Euclid’s Elements*, 267–68.

8 Dale Jacquette, *David Hume’s Critique of Infinity* (Leiden-Boston-Köln: Brill, 2001), 179.

9 A line segment can be bipartitioned if it can be divided into two equal parts which are separated in space. The bisection of a line segment is obtained when the line segment is crossed by another line exactly at its midpoint.

10 Mark Pressman, "Hume on Geometry and Infinite Divisibility in the *Treatise*," *Hume Studies* 23.2 (1997): 227–44.

11 Lorne Falkenstein, "Hume's Finite Geometry: A Reply to Mark Pressman," *Hume Studies* 26.1 (2000): 183–85.

12 It is very likely that Falkenstein himself did not intend to suggest that the above mentioned strategy is how Hume would answer Pressman's objection; rather, he meant to point out a flaw in that objection.

13 This is also one of the basic tenets of more recent trends in geometry (such as projective geometry) which take distance to be a relative notion rather than a quantity measured in terms of an absolute unit of measure (such as the indivisibles).

14 A list of the five equality axioms (or, as they are also called, common notions) can be found in *The Thirteen Books of Euclid's Elements*, 155. See also Thomas L. Heath's commentary on the fourth common notion, *ibid.*, 224–31. In the *Abstract* of the *Treatise* (T Abs. 29; SBN 659), Hume appears to subscribe to the idea that "the word [equality] admits of no definition," which suggests that he is only concerned with how one can test equality.

15 The relevant paragraphs from Isaac Barrow's *Lectures* are reprinted in Smith, *The Philosophy of David Hume*, 343–46.

16 It is true that we derive our idea of extension from the idea of an indivisible. However, this is not the standard from which we derive our ideas of a right line.

17 I take it that Hume uses "exact" and "precise" interchangeably and that he applies them to both ideas and statements.

18 Proofs (or derivations) should be distinguished from demonstrations. A demonstration is a truth-preserving proof.

19 Some commentators attribute to Hume a general criticism of geometry's claim to certainty as well as the idea that "the minima of perception are far too elusive to provide a good foundation for demonstrative knowledge." Marina Frasca-Spada, *Space and the Self in Hume's Treatise* (Cambridge University Press, 1998), 129–30. This would create a tension with other paragraphs where Hume considers geometry to be a part of demonstrative knowledge, which Frasca-Spada tries to explain by a rather contrived distinction between two types of certainty (as she would put it, "a demonstrative argument may be certain, and yet we may not be certain about it" [Frasca-Spada, 154]). According to my interpretation, there is no tension of this sort: geometry is, for Hume, a source of certainty and demonstrative knowledge when it is not concerned with extremely small entities.

20 In particular, on "observing the physical properties of diagrams" (Fogelin, "Hume and Berkeley," 57).

21 Fogelin tries to explain the failure of the proof of infinite divisibility by saying that "a priori proofs of infinite divisibility do not show that *physical* (emphasis mine) space is infinitely divisible" ("Hume and Berkeley," 57). This would not explain why for Hume the proof fails to establish that the idea of a line segment is infinitely divisible.

22 One might argue that, strictly speaking, the equality asserted by the Pythagorean theorem is not about the equality of line segments, but the equality of some numbers.

Hume agrees that we do have a precise standard for numbers. However, given that the idea of a line segment is not an exact idea, the number associated with it can be only a rough estimate of its relative length. Moreover, one can think of the theorem as proving that the area of the square determined by the hypotenuse and the joint area of the squares determined by the other two sides are equal.

23 The restriction is needed to exclude proofs of the Pythagorean theorem which might explicitly appeal to extremely small entities. The most famous proofs, including Euclid's proof I.47 in the *Elements* do not rely on entities of this sort.

24 Admittedly, some things Hume says, such as his remark that geometry is "built on ideas, which are not exact, and maxims, which are not precisely true" (T 1.2.4.17; SBN 45), might suggest that he thinks of the theorems of geometry as mere approximations that fall short of being true. I think that the correct reading of Hume's remark is not that the maxims are only approximately true, but that they are truths which deal with the imprecise standard of equality. This is the reading that is consistent with both the remarks from the *Treatise* quoted in the first section of this paper and the claim made in the *Enquiry* (EHU 4.1; SBN 108) that "the truths demonstrated by Euclid would for ever retain certainty and evidence."

25 I am indebted to an anonymous reviewer for emphasizing the importance of clarifying this distinction.

26 See Thomas L. Heath's commentary in *The Thirteen Books of Euclid's Elements*, 268.

27 I thank an anonymous reviewer for a very helpful discussion of the theorems and axioms that are not statements of equality.

28 In the *Abstract* of the *Treatise*, Hume claims that "all Geometry is founded on the notion of equality and inequality" (T Abs. 29; SBN 658).

29 See Hume's discussion in T 1.2.4.30 (SBN 51). He dismisses, perhaps a bit too hastily, the possibility of denying that two lines could appear both as separated by an inch and as having a segment in common.

30 Thus, it seems possible for one to articulate an objection based on a sorites argument. A series of imprecise equalities might entail an obviously false equality claim. There are also some familiar objections against the notion of vague identity (the standard source is Gareth Evans's "Can there be Vague Objects?" *Analysis* 38 [1978]: 208) which could be adapted to the imprecise notion of identity suggested in the previous paragraph. Although these are indeed serious objections that would have to be addressed in an expanded version of the view, the arguments they are based on are controversial. Moreover, Hume might be able to defend himself by saying that these difficulties are not specific to geometry but to any discipline which operates with vague notions.

31 Smith, *The Philosophy of David Hume*, 530–36, offers a useful discussion of Hume's attitude to the *Treatise*.