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Turnabout is Fair Play: A New Humean Response in the Old Debate with Kant

PETER THIELKE

Abstract: Kant claims that Hume failed to see that mathematics provides us with synthetic a priori knowledge; had he done so, Kant argues, Hume would have to admit the possibility of such knowledge in causal judgments as well. Instead, Kant insists that Hume treats mathematics as analytic, and so missed the key insights of the Critical philosophy. I argue that it is rather Kant who is mistaken: Hume, in fact, endorses a position very similar to the view that mathematics is synthetic and a priori, and arrives at an account of mathematical necessity that stands as a plausible alternative to Kant's. More importantly, recognizing this Humean account of mathematics exposes a potentially grave vulnerability in Kant's system that Hume might exploit: while mathematics can be seen as synthetic a priori knowledge, Hume can argue that this gives us good reason to think that causal judgments cannot meet this standard of necessity.

Kant famously noted that a memory of Hume “interrupted” his dogmatic slumbers, an alarm commonly taken to have been sounded by the challenge Hume raised against the rational foundations of causal connections. The *Prolegomena*'s discussion of the role played by Hume's skepticism in the development of the critical philosophy makes it relatively easy to see how to formulate something along the lines of “Kant's response to Hume,” and a great deal of ink and toil have been devoted to debates about the success of this project. Developing “Hume's

response to Kant” is a bit trickier, not least because by the time Kant’s views were made available, Hume had succumbed to a slumber much more permanent and literal than the dogmatic variety endured by Kant.

Given the centrality of causation in both Hume’s own work and Kant’s response to it, we might be tempted to seek to have Hume engage Kant directly, by challenging the transcendental account of causality developed in the first *Critique*, or by insisting that Kant cannot solve the skeptical problems about particular causes that Hume raised in both the *Treatise* and the *Enquiry*.¹ On this line of thought, while Kant might be able to show that the principle that every event has a cause is rationally grounded, this does not suffice to rebut Hume’s claims that we cannot establish the rational foundation on which we can know that the same causes will have the same effects. Hume’s response to Kant would, in effect, be just to reiterate the same arguments he makes against *all* of the broadly “rationalistic” accounts of causation on offer.² This, I take it, has been the strategy adopted by many of Hume’s defenders, and it is reflected, I suspect, in the attitude taken by Kant’s partisans as well—both sides see the status of causal connections as the crucial issue in the dispute between the two figures.

Unfortunately, the preoccupation with causation has led to entrenched positions on both sides, and a seeming impasse about how to proceed. In the hopes of providing a bit of a nudge to the debate, I plan to sketch an alternative way in which Hume might respond to Kant, focusing not on causal claims but rather on the status of mathematical cognition. I will suggest that one of Kant’s key cudgels against Hume—the claim that mathematics rests on synthetic a priori judgments—can in fact be redeployed by Hume *against* the critical philosophy, in a bit of philosophical jiu-jitsu. For Hume, as we will see, turnabout is fair play, and it leads to an interesting and novel objection to Kant.

I. The Status of Mathematics

While Kant holds Hume’s central challenge to philosophy to be his presumed skepticism, it is interesting that Kant takes Hume to have at least glimpsed the problems—and even the possible solutions—addressed in his own critical philosophy. In the *Prolegomena*, for instance, Kant notes that Hume proved that “it is simply not to be seen how it could be, that because something is, something else necessarily must also be, and therefore how the concept of such a connection could be introduced a priori.”³ And, in the B-Introduction to the first *Critique*, Kant remarks that while Hume “came closest” to the problem of explaining how synthetic a priori judgments are possible, he could not see how to ground the necessity of such claims in reason, and instead resorted to custom and habit.⁴

The roots of Hume’s inability—or perhaps unwillingness—to locate a rational foundation for causality, however, are for Kant found not so much in his analysis

of causation itself as in his failure to recognize the threat that his position presents to mathematics. In the *Critique*, Kant argues that Hume would never have rejected the synthetic a priori had he been aware that this would mean that “there could also be no pure mathematics, since this certainly contains synthetic a priori cognitions, an assertion from which his sound understanding would surely have protected him” (B20).⁵ The same point is echoed in the *Prolegomena*, where Kant asserts that Hume essentially held that mathematics rested

solely on the principle of contradiction; and although he had by no means made a classification of propositions so formally and generally, or with such nomenclature, as I have here, it was nonetheless just as if he had said: Pure mathematics contains only analytic propositions, but metaphysics contains synthetic propositions a priori. Now he erred severely in this, and this error had decisively damaging consequences for his entire conception. For had he not done this, he would have expanded his question about the origin of our synthetic judgments far beyond his metaphysical concept of causality and extended it also to the possibility of a priori mathematics; for he would have had to accept mathematics as synthetic as well. But then he would by no means have been able to found his metaphysical propositions on mere experience, for otherwise he would have had to subject the axioms of pure mathematics to experience as well, which he was much too reasonable to do. (4:272–73)

Such an accusation even makes an appearance in the *Critique of Practical Reason*, where Kant again claims that Hume erroneously “held that [mathematics’] propositions were all analytic,” and as a result failed to see that his attacks on reason would also have the malign effect of producing “skepticism even with respect to mathematics and consequently in every *scientific* theoretical use of reason.”⁶

In all its guises, Kant’s argument is relatively straightforward: had Hume the benefit of Kant’s insight that mathematics is a synthetic a priori discipline, he would have been forced to recognize that such judgments must be possible, and this would have opened the door to including causality in such an account. But since Hume took mathematics to be concerned only with analytic judgments, and governed solely by the principle of contradiction, he failed to see the pressing need to ground synthetic a priori judgments more generally, and instead mistakenly dispensed with them entirely.

The idea that the critical system would have neatly unfolded before Hume’s eyes had he only seen the synthetic character of mathematics is a bit outlandish, although Kant’s general point does seem to be on the mark: at the very least, the fact that the logical space of the synthetic a priori is not empty could have given Hume pause before rejecting the possibility of a rational foundation for causality.

But Kant's accusation really only finds its target if Hume did indeed take mathematics to be an analytic discipline, and it is not at all clear that this is really the case. One question, then, is whether Kant is fairly depicting Hume's views. But there is a bigger issue afoot here, since it is precisely the charge that Hume neglected the synthetic nature of mathematics that can potentially be turned against Kant. Hume, I will propose, not only can accept that mathematics is in some sense synthetic and a priori, but can also leverage this claim to drive a wedge in Kant's own account of causality specifically, and necessity more generally.

II. Hume on Mathematics

Unlike many of the other towering figures of modern philosophy, Hume was neither especially well versed, nor seemingly even terribly interested, in mathematics.⁷ The *Enquiry* has only several mentions of the topic, and while the *Treatise* does contain an extended discussion of geometry, this is largely advanced in the service of making Hume's case against infinite divisibility, rather than as an independent analysis of the foundations of geometrical knowledge.⁸ Why then did Kant think that Hume endorsed an analytic account of mathematics?

The answer can likely be found in the famous distinction Hume draws between relations of ideas and matters of fact at the beginning of section 4 of the *Enquiry*. Geometry, algebra and arithmetic are described as being based on relations of ideas, in which "every affirmation . . . is either intuitively or demonstratively certain." And, Hume continues, in these subjects, "propositions of this kind are discoverable by the mere operation of thought, without dependence on what is anywhere existent in the universe. Though there never were a circle or triangle in nature, the truths demonstrated by Euclid would for ever retain their certainty and evidence" (EHU 4.1; SBN 25). Matters of fact, by contrast, depend upon experience, and lack the demonstrative certainty enjoyed by relations of ideas. The descriptions of the two forms of judgments seem to map neatly onto Kant's own distinction between analytic and synthetic judgments, and it is natural to think that this assumption led Kant to believe that Humean relations of ideas—and specifically mathematical propositions—are essentially analytic claims.

This has been a standard way of interpreting Hume's Fork,⁹ but a bit of reflection suggests that it sits uneasily with the positions of both the *Enquiry* and the *Treatise*. While it is true that Hume holds that relations of ideas "are discoverable by the mere operation of thought," this by itself does not establish that such truths are analytic in Kant's sense, since such an attribution would require a further claim that certain ideas are "contained" in others as something like definitional marks.¹⁰ But Hume's examples in the *Enquiry*—the Pythagorean Theorem and the proposition that three times five is equal to the half of thirty—need not be taken to express analytic statements, at least not as Kant uses the term.¹¹ Rather, they

are perhaps better understood as being *a priori* claims, which is underscored by Hume's assertions that they do not depend upon the existence of objects in the world.¹² The truths of mathematics, on this line of thought, are necessary because their opposite cannot be conceived without a contradiction, and this shows, Hume argues, that demonstrations—which establish necessary conclusions—are possible in arithmetic and algebra.

This by itself might quickly settle the issue of whether Hume takes mathematics to be analytic, since one can see contradiction as marking out Kant's notion of analyticity: a judgment is analytic if its opposite cannot be thought without contradiction, where this is taken as a necessary and sufficient condition. Even though "all events have a cause" is offered up by Kant as a synthetic *a priori* proposition, it seems that we can think of uncaused events without contradiction; by contrast, in the analytic judgment "all bodies are extended," a contradiction would arise in trying to think of bodies apart from extension. So, it might be argued, since Hume claims that the opposite of relations of ideas involve a contradiction—which, on this line of thought, is just Kant's test of analyticity—it clearly follows that Hume would view mathematics as analytic, since the realm of relations of ideas is essentially exhausted by algebra and geometry.

Hume is typically viewed in this way, but the quick argument on offer here is too hasty. Putting aside the question of whether only the opposites of analytic judgments involve a contradiction,¹³ the suggestion that Hume is obviously committed to the analyticity of mathematics turns on assuming that a Humean contradiction is equivalent to Kant's notion, but this is far from clearly being the case. For Kant, the contradiction that comes in trying to think of an unextended body, for example, arises because extension is part of the set of definitional marks that make up the concept of "body," but this position sits very uneasily with Hume's essentially *imagistic* account of mental contents. Hume claims that if something is "demonstratively false, it would imply a contradiction, and could never be distinctly conceived by the mind" (EHU 4.2; SBN 26), but by conception Hume does not mean anything like Kant's appeal to definitional marks; rather, for Hume, conception is essentially an act of imagination, in which we attempt to "present" various ideas as imagistically connected to one another. And, if this is the notion of contradiction at stake, it is not at all clear that Kant would allow that we can conceive the opposite of synthetic *a priori* judgments without giving rise to this kind of Humean contradiction. At issue is not just the mere thought of a proposition, but how it can be cognized by us, according to the conditions on possible representation. As L. W. Beck puts the point, we can "think without contradiction an uncaused event; hence the relation expressed in the judgment is synthetic; but we cannot imagine, represent, or know an event as objective without first relating it to another event by a rule of causation; hence the judgment is known *a priori*" ("Kant's Synthetic Judgments," 309). Given this broader Humean

notion of contradiction, in which to conceive the opposite of a judgment is to *imagine* or *represent* it—and not merely think it, in Kant’s usage of the term—the fact that Hume claims that the opposites of relations of ideas cannot be conceived without contradiction does not by itself settle the question of whether they are analytic, in the sense Kant proposes.¹⁴ Can they then be seen as synthetic, or as at least resembling what Kant means by the synthetic?

For Kant, synthetic judgments always involve intuitive content, which is either received from the senses, or, in the case of mathematics, constructed in pure intuition. The general idea is that in order to cognize a synthetic judgment, an analysis of the concepts involved will alone not suffice: to take Kant’s famous example, we cannot know that $5 + 7 = 12$ simply by inspecting the concepts “5,” “7,” and “12,” or by appealing only to their definitions. Rather, to show that $5 + 7 = 12$, a mathematical *operation* must be performed—we must actually add up 5 and 7—and this, Kant proposes, requires an intuitive element: as he puts it, “without getting help from intuition we could never find the sum by means of the mere analysis of concepts” (B16).

Surprisingly, Hume makes almost exactly the *same* point in the *Enquiry*, where he notes that

It seems to me that the only objects of the abstract sciences or of demonstration are quantity and number, and that all attempts to extend this more perfect species of knowledge beyond these bounds are mere sophistry and illusion. As the component parts of quantity and number are entirely similar, their relations become intricate and involved; and nothing can be more curious, as well as useful, than to trace, by a variety of mediums, their equality or inequality, through their different appearances. . . . *That the square of the hypotenuse is equal to the squares of the other two sides, cannot be known, let the terms be ever so exactly defined, without a train of reasoning and enquiry.* But to convince us of this proposition, that where there is no property there can be no injustice, it is only necessary to define the terms, and explain injustice to be a violation of property. This proposition is, indeed, nothing but a more imperfect definition. It is the same case with all those pretended syllogistic reasonings, which may be found in every other branch of learning, *except the sciences of quantity and number*; and these may safely, I think, be pronounced the only proper objects of knowledge and demonstration (EHU 12.3; SBN 163, emphases added).

The contrast here between mathematical demonstrations and definitional equivalences mirrors, I propose, Kant’s distinction between synthetic and analytic judgments, and it strongly suggests that, since definitions of terms are *not*

sufficient to secure mathematical knowledge, Hume would likely *agree* with Kant that mathematical demonstration requires something akin to intuitive content, which in turn suggests that Hume would follow Kant in thinking that mathematics is synthetic. It is important to note that the claim is *not* that Hume anticipates Kant's specific sense of intuitive content, or that we should try to find in Hume anything like a Kantian intuition grounded in the pure forms of sensibility. The point is only that Hume, like Kant, sees mathematical judgments as requiring some kind of counting or calculation, which must go beyond merely considering the definitions of the terms defined.¹⁵ I will argue that while both Kant and Hume see the need for this "calculative" step—indeed, this is what makes mathematics synthetic for both of them—each will give a different account for what can serve in this role. But, if this is the case, then Kant's charge that Hume failed to see the synthetic nature of mathematics looks like it, in fact, misses the mark.¹⁶

A similar point also can be found in the discussion of space and time in the *Treatise*, where—in the context of arguing against infinite divisibility—Hume approvingly cites Joseph Butler's view that "all definitions . . . are fruitless, without the perception of such objects [taken to be identical]; and where we perceive such objects we no longer stand in need of any definition. To this reasoning I entirely agree; and assert, that the only useful notion of equality, or inequality, is deriv'd from the whole united appearance and the comparison of particular objects" (T 1.2.4.22; SBN 637). Likewise, Hume claims that while mathematicians pretend to give a precise definition of a right line when they say it is the shortest between two points, this is "more properly the discovery of one of the properties of a right line, than a just definition of it. . . . A right line can be comprehended alone; but this definition is unintelligible without *comparison* with other lines, which we conceive to be more extended" (T 1.2.4.26; SBN 50, emphasis added). This presages the position of the *Enquiry* and further belies Kant's accusation that in taking mathematics to traffic only in relations of ideas, Hume consigned it to being an analytic discipline: in order to grasp equalities, for example, we need more than just a definition, but instead require some kind of sensible content or "united appearance."

Finally, the suggestion that Hume does not endorse an analytic or definitional account of mathematical is also supported by a claim made in the *Treatise* about abstract ideas. There, Hume notes that were we to hear the word "triangle," and as a result form an idea of an equilateral three sided figure and assert that "the three angles of a triangle are equal to each other," the falsity of this claim would not arise from noting anything about the definition of a triangle, but would rather come about when "the other individuals of a scalenum and isosceles, which we overlook'd at first, immediately crowd in upon us, and make us *perceive* the falsehood of this proposition" (T 1.1.7.8; SBN 21, emphasis added). The counterexamples to the faulty claim about the nature of a triangle's angles are not the products of

definitions, but again involve something like intuitive content: we *see* that a scalene or isosceles triangle does not have equal angles, rather than infer this from how a triangle is defined.

On Kant's view, synthetic judgments require something more than just conceptual analysis, but for Hume, what could provide this additional content? According to Kant, the necessity of mathematical judgments arises from the specific possibility of construction in *pure* intuition,¹⁷ yet the notion of a pure form of sensibility that allows for such construction seems far removed from Hume's generally empiricist position. Perhaps, then, even if Kant is wrong to claim that Hume views mathematics as analytic, he can still try to pin him on the horns of a dilemma: Hume can have mathematics be *a priori* or synthetic, but not both. Since it seems that Hume does not have room for construction in pure intuition, the intuitive content of mathematical judgments would have to be empirical, in which case necessity would be lost; if Hume wants mathematics to be necessary, then he would have to take it as analytic. Kant's initial accusation might be too hasty, but on this line of thought it finds a target nonetheless.

It would be foolish to pretend that Hume anticipates Kant's notion of pure intuition, so if he is to avoid the charge leveled at him, some different account of the intuitive content of mathematical judgments—one that still preserves their necessity—must be found. Clearly we cannot base mathematical truths on merely empirical content and yet lay claim to necessity, but fortunately for Hume, there is at least the possibility that aspects of empirical content can nonetheless contain elements that would suffice to ground mathematical necessity, in a way that—perhaps—could avoid Kant's dilemma.

The initial step of this admittedly conjectural claim is found in the discussion of distinctions of reason in the *Treatise*. This comes up in the context of Hume's account of abstract ideas, and concerns the way in which we are able to consider properties as distinct, even though they are always represented together. To take Hume's example, we cannot "separate and distinguish" the color and shape of a white marble globe when we perceive it, since "the mind would never have dreamed of distinguishing a figure from the body figured" (T 1.1.7.18; SBN 25). But while we cannot perceive any difference between the color and shape of the globe, we can still form independent ideas of its hue and form, when we note, say, a black sphere or a white cube. In such cases, Hume claims, "we begin to distinguish the figure from the color by a distinction of reason; that is, we consider the figure and color together, since they are in effect the same and indistinguishable; but still view them in different aspects, according to the resemblances, of which they are susceptible" (T 1.1.7.18; SBN 25). When we make a distinction of reason, we can consider or view the same sensible content under a different description, or note features of it that cannot be perceptually represented independently of the various other properties that an object might possess.

The suggestion then is that mathematical judgments rest on properties that have been arrived at through a distinction of reason. In the judgment that, say, $5 + 7 = 12$, the ideas of the numbers are not formed simply by assembling collections of beans, beads, or bears, and counting them up; rather, having seen a variety of groups of things, we can form an idea of a particular number through a distinction of reason, much as we can form an idea of a sphere by seeing differently colored globes. We do not get a separate and distinct impression of numbers apart from our perceptions of things—there is no distinct impression of 5, except in so far as we see five fingers, five toes and so on—but we can still attend to the shared quality that each set of objects has, and to note that each represents the idea of 5.¹⁸

This does not mean that in every case we will have complete grasp of the component parts of all numbers—as Hume notes, with a large number such as 1,000 we have “generally no adequate idea of it”—but the mind does have the “power of producing such an idea, by its adequate idea of the decimals, under which the number is comprehended” (T 1.1.7.12; SBN 23). The idea of 1,000 does not—and likely cannot—arise from considering sets of a thousand objects, but rather is formed by compounding adequate ideas of ones and tens to produce the idea of hundreds, which in turn can be combined to create the idea of 1,000. Despite the fact that we cannot have an adequate idea of large numbers, Hume insists that “this imperfection . . . in our ideas, is never felt in our reasonings” (T 1.1.7.12; SBN 23): we can draw sound conclusions about the sum of 12,345 and 6789 without having adequate ideas of either one of the addends taken alone.¹⁹

Although he does not make this point explicitly, Hume’s claims that our mathematical reasoning is not threatened by the fact that in many cases we do not have an adequate grasp of numbers suggest a crucial distinction between the ideas of individual numbers on the one hand, and the *relations* between them on the other, and it is here that we might hope to locate the ground of mathematical necessity. Drawing on a proposal by Dorothy Coleman, we can see Hume as marking a distinction between the empirically derived ideas of particular numbers, and the a priori and necessary relations that hold *between* them.²⁰ The suggestion is that while “5,” “7,” and “12” are all empirically produced ideas, the relation “ $5 + 7 = 12$ ” is not, but rather enjoys the status of being a priori. This necessity rests on the relation of *proportion* in quantity or number, which provides us

with a precise standard, by which we can judge of the equality and proportion of numbers; and according as they correspond or not to that standard, we determine their *relations*, without any possibility of error. When two numbers are so combined, as that the one has always an unite answering to every unite of the other, we pronounce them equal (T 1.3.1.5; SBN 71, emphasis added).

In the terms Hume employs in the *Enquiry*, the opposite of “ $5 + 7 = 12$ ” is inconceivable because it is unimaginable—we cannot even form a coherent idea of $5 + 7$ not equalling 12. But the reasons why this is the case turn not on the way that the elements of the proposition are defined, but rather on how they are constructively linked to one another in the judgment, by way of what Owen calls intermediate ideas.²¹ And, while the component parts of arithmetic judgments—the individual numbers formed from uniform units—might have an empirical origin, once we have them in our possession the relations between them can be taken to be a priori. This, I suggest, is what Hume means when he claims that “propositions of this kind are discoverable by the mere operation of thought, without dependence on what is anywhere existent in nature” (EHU 1.4.1; SBN 25). He is here noting the *a priori*, and not analytic, character of mathematical judgments.

The notorious discussion of geometry in the *Treatise* also provides some support for the view that Hume takes mathematical relations to involve something resembling synthetic a priori propositions. For Hume, while geometry approximates to the standards of certainty, it fails to reach them because there can be no rigorous criterion for the equality of lines, since the “prodigious minuteness” of nature prevents us from perceiving whether the component parts of lines stand in the one-to-one relationship that equality demands. Instead, algebra and arithmetic are “the only sciences, in which we can carry on a chain of reasoning to any degree of intricacy, and yet preserve a perfect exactness and certainty,” since in this case we can meet the “precise standard” that equality and proportion demand (T 1.3.1.5; SBN 71). Unlike the other “certain” relations of resemblance, contrariety and degrees, those of proportions in quantity or number—the foundation of mathematics—are not immediately grasped, but rather fall under the province of demonstration, which in turn suggests something akin to Kant’s requirement that synthetic propositions in mathematics involve some kind of calculation or synthetic operation. In providing mathematical demonstrations, then, we engage with the *relations* between ideas, and note that the elements of, say, “ $5 + 7$,” are perfectly congruent with the elements of “12.” The crucial point here is that the necessity that holds of the proposition “ $5 + 7 = 12$ ” arises not from the ideas considered singly, but rather in the relation that obtains between them. As Hume claims, there is nothing distinctive about the ideas of numbers that sets them apart from other ideas—they have no “spiritual nature” and do not require “superior faculties of the soul” to grasp them (T 1.3.1.7; SBN 72)—*except* that they are capable of meeting the standards that are presented by the relation of identity. The necessity then is not “in” the numbers, as it were, but rather in the way that they are related to each other in the mathematical propositions under consideration. As Hume puts the point, “the necessity, which makes two times two equal to four, or three angles of a triangle equal to two right ones, lies only in the act of the understanding, by which we consider and compare these ideas” (T 1.3.14.23; SBN 166).

It is still not clear, though, that this account of Hume's mathematical judgments rises to the level of "necessity and strict universality" (B4) that Kant thinks indicates a priori cognition. Indeed Kant notes that such strict universality would be lost if one attempts, "as Hume did, to derive it from a frequent association of that which happens with that which precedes and a habit (thus a merely subjective necessity) of connecting representations arising from that association" (B5). It is important to keep in mind, though, that Kant here has Hume's account of causation in his sights—which, to be sure, is based only on what seem to be subjective associations—and it need not follow that Hume's treatment of mathematical judgments also only can lay claim to mere subjective necessity. In fact, a distinction Hume makes concerning the notion of truth suggests that this account of mathematical judgments *can* meet—or at least approach—the conditions on a priori cognition that Kant demands.

Truth, Hume notes at the end of Book 2 of the *Treatise*, consists either "in the discovery of the proportions of ideas, consider'd as such, or in the conformity of our ideas of objects to their real existence" (T 2.3.10.2; SBN 448). The former method rests on demonstrative proofs, while the latter is only sensible, and the emphasis on the demonstrable *proportion* of ideas—that is, their relations to each other—rather than to the ideas themselves again suggests the kind of view developed above. What is at issue are not subjective associations and habits of mind, but rather the proportions "consider'd as such," which certainly hints at the kind of absolute necessity that Kant demands of a priori cognition. Demonstration, which presumably provides us with something like Kant's notion of a priori truths, rests not on contingent ideas alone, but instead on the way that they are necessarily related to one another. This relation, moreover, does not simply express an association of ideas that rests on the merely subjective states of an observer. There is nothing necessary about "5," "7," and "12" taken singly, but the judgment " $5 + 7 = 12$ " is itself a priori.

Does all of this allow Hume to avoid the Kantian objection that he can find no place for synthetic and a priori judgments in mathematics? Since Hume is of course not directly operating within the confines of Kant's distinctions between analytic and synthetic, and a priori and a posteriori judgments, some further speculation is required, though I think that there are some tantalizing hints that support the claim that Hume can manage a response to Kant's dilemma. While Hume does not directly make this claim, a passage in the *Treatise* suggests that he is at least amenable to the idea that mathematical ideas, which are based on relations, enjoy something like a "pure" or even a priori status.²² In responding to the defender of infinite divisibility, Hume notes that

I know there is no mathematician, who will not refuse to be judg'd by the diagrams he describes upon paper, these being loose drafts, as he will tell

us, and serving only to convey with greater facility certain ideas, which are the true foundation of all our reasoning. This I am satisfy'd with, and am willing to rest the controversy merely upon these ideas (T 1.2.4.33; SBN 53).

Hume does not elaborate on what precisely the nature of these ideas “which are the true foundation of all our reasoning” involves, but it seems plausible—especially given the contrast between them and mere marks on paper—to treat them as something like “pure” mathematical ideas.²³ Likewise, in the discussion of truth at the end of Book 2 of the *Treatise*, Hume distinguishes between “a mathematical demonstration” that discovers the equality of two figures, and the “sensible” method of using two compasses to measure the same shapes. In the former, “the proofs be demonstrative, and in the other only sensible, yet generally speaking, the mind acquiesces with equal assurance in the one as in the other” (T 2.3.10.2; SBN 449). We reach the same conclusion about the equality of the figures whether we prove it mathematically, or measure it mechanically, but the paths that each method takes are clearly distinguished. At the very least, this hints that Hume would likely have no principled objections to a priori mathematical judgments, and would classify them differently from merely “sensible” judgments. While the latter might only achieve the kind of subjective necessity that Kant disparages, the former can rise to the level of absolute necessity that Kant thinks characterizes mathematical judgments.²⁴

But, given Hume’s incessant demand to locate the impression that gives rise to any idea, what could serve in this role for such a priori content? Here again Kant’s dilemma might return, since if we need an *impression* of a logically necessary relation, then it is not clear how we can retain the a priori nature of mathematics; however, if no such impression is needed, and mathematics is grounded only on the “operations of thought,” which in turn are guided solely by the principle of non-contradiction, then it seems that the synthetic character of mathematics would be lost. The response on Hume’s behalf, I propose, must be to split the difference, and argue that while impressions are indeed needed, they do in fact provide us with necessary relations, and demand more than simply non-contradiction.

In order to make this case, it is important to note that on the view I have sketched, Hume does not require that we have an independent impression of mathematical necessity—it is not as if, in addition to the relation between, say, $5 + 7$ and 12 there is a separate impression of the necessary connection between them. Rather, the ideas of $5 + 7 = 12$ and the necessary relation between are, in the terms of the *Treatise*’s discussion of distinctions of reason, “in effect the same and indistinguishable”; they are “different aspects, according to the resemblances, of which they are susceptible” (T 1.1.7.18). In this sense, the necessity present in mathematical judgments is similar to the idea of time, which, Hume claims “is

not deriv'd from a particular impression mix'd up with others, and plainly distinguishable from them; but arises altogether from the manner, in which impressions appear to the mind, without making one of the number" (T 1.2.3.10).

Hume's view of mathematics more closely approaches Kant's position when the latter is understood along the lines proposed by Lisa Shabel, who convincingly argues that Kant's account of the mathematical construction of a priori intuition does not involve the creation of a wholly abstract entity, but rather is a way of treating an empirically drawn figure in a "pure" manner. As she puts it, "the pure intuitions which exhibit and construct mathematical concepts, and on which mathematical demonstrations are based, are intuitions of single, individual, sensible objects *considered* in conjunction with the procedure for the construction of those objects" (94, emphasis added).²⁵ When the mathematician constructs an object in pure intuition, on this view, she creates an empirical image, but only attends to the "pure" elements of the diagram: as Kant puts it, "the individual drawn [mathematical] figure is empirical, and nevertheless serves to express the concept without damage to its universality, for in the case of this empirical intuition we have taken account only of the act of constructing the concept" (A714/B742). This closely mirrors Hume's assent to the mathematician who insists on a difference between the physical marks on the paper, and the mathematical relations they convey: for both Kant and Hume, it seems, a distinction of reason allows for the same object to be treated either empirically or purely, depending on which elements of it are considered.

The distinctive "manner" in which mathematical relations appear to the mind is then reflected in the "perfect exactness and certainty" characteristic of algebra and arithmetic, and presumably the operations of thought that establish these relations of equality and proportion provide the necessity of mathematical judgment. But this need not mean that Hume must be impaled on the other horn of the dilemma, which holds that mathematics is governed only by the principle of contradiction, since on Hume's view this is only a necessary, but not sufficient, condition on mathematical reasoning. While Hume does claim that "if any term be defined in geometry, the mind readily, of itself, substitutes, on all occasions, the definition for the term defined"—which might suggest that mathematics is only definitional, and as a result analytic—he continues by noting that "even when no definition is employed, the object itself may be presented to the senses, and by that means be steadily and clearly apprehended" (EHU 7.1; SBN 60). In mathematics, it seems, definitions are dispensable, but this in turn would imply that the principle of non-contradiction alone is not sufficient for mathematical reasoning—we instead need the kind of sensible content discussed above.

Whether this account of mathematics in terms of the distinctions of reason is ultimately enough to secure Hume a safe passage between the horns of Kant's dilemma remains a controversial subject. Henry Allison, for example, argues that

Hume can do so only by tacitly appealing to the notion of a concept as a rule of recognition, which he thinks Hume needs in order to explain how we can recognize the relevant mathematical or numerical features among various sets of objects (*Custom and Reason*, 85–86). But, since Hume’s official empiricist and imagistic position seems to have no place for such view of concepts, the account on offer here might require a good deal of textual and interpretive contortions. While these remain pressing worries, however, it seems that there is at least a plausible way in which we can see Hume as allowing for—and even endorsing—mathematics as a synthetic a priori discipline, in a way that bears some clear similarities to Kant’s own view. At the very least, Kant’s initial accusation that Hume simply missed the synthetic a priori nature of mathematics should itself be viewed with some suspicion.

III. The Twist

This is all well and good, but similar points have been made before.²⁶ The further, unexplored issue is what lessons we should draw from Hume’s possible response to Kant’s charges. Most commentators have been content simply to show that Hume need not be saddled with Kant’s claim that he reduces mathematics to an analytic discipline. What I wish to suggest is that seeing Hume as in rough agreement with Kant about the status of mathematics has an unnoticed—though potentially grave—implication for the Critical philosophy. For if Hume can accept that mathematics is synthetic a priori, he can perhaps turn this fact *against* Kant: Hume might paraphrase the earlier accusation in the *Critique*, and hold that ‘Had *Kant* only seen the real nature of mathematical synthetic a priori knowledge, he would never have thought that this could apply also to causal judgments, and would have recognized that we must sharply distinguish the two, and limit synthetic a priori judgments *only* to mathematics. Now he erred severely in this, and this error had decisively damaging consequences for his entire conception.’

The general idea is that Hume can claim that there is a fundamental *difference* between how we secure synthetic a priori knowledge in mathematics, and how we might do so with regard to causal judgments, and that recognizing this fact should lead us to be suspicious of Kant’s account of causality. In Hume’s terms, the distinctions of reason we use to extract numbers from sets of empirical objects produce ideas that can stand in relations of necessity, and this is what guarantees their synthetic a priori nature. But the same procedure *cannot* be followed with causal judgments, since the relations in which ideas stand with regard to causal connections do not enjoy the same necessity; while the relation of proportion in number preserves “a perfect exactness and certainty” (T 1.3.1.5; SBN 71), the relation of cause and effect does not. Kant accused Hume of not seeing that the synthetic a priori nature of mathematics opened the door to a more general

notion of the synthetic a priori status of causality, but Hume can turn this around by arguing that in fact, there is only *one* route to a priori synthetic knowledge, and it must go through mathematics alone—and noting this fact *excludes* precisely the causal judgments Kant sought to let in.²⁷ Intuitive or demonstrative certainty—the hallmark of the category of relations of ideas found in section 6 of the *Enquiry*—is then for Hume also *limited* to the “sciences of Geometry, Algebra and Arithmetic” (EHU 4.1.1; SBN 25), and it is a mistake to think that it can also be found in causal judgments.

Indeed, Hume might continue, the problem is even more pronounced for Kant, since his account of synthetic a priori knowledge in mathematics turns crucially on the possibility of construction in pure intuition, which Kant claims is inapplicable in the case of causality. Why, Hume might ask, should we think that allowing for the synthetic a priori nature of mathematics tell us *anything* about the possibility of synthetic a priori causal judgments, since the two have seemingly wholly different structures? In the “Discipline of Pure Reason” in the first *Critique*, Kant sharply distinguishes the constructive methods of mathematics from the discursive procedures of philosophy, and notes that though the two disciplines “have a common object, the manner of dealing with it through reason is entirely different in philosophical than in mathematical consideration” (A715/B743). But why need Hume accept that there is a common object between mathematics and philosophy, especially if, as we have seen, he would allow only one, “arithmetic” path to necessary knowledge?²⁸ Later in the “Discipline,” Kant claims—in direct response to Hume’s skepticism—that “we are really in possession of synthetic a priori cognition” (A763/B791), but Hume would likely accept this only with regard to mathematics, and insist that the burden is on Kant to show how we can also secure synthetic a priori cognition in the realm of experience, in a way different than is possible in mathematics. Given the distinctive constructive methods that Kant proposes for mathematics, it is open for Hume to insist that these tell us nothing about causation, or the possibility of finding synthetic a priori knowledge in science.

This, of course, does not settle the question in Hume’s favor, since Kant’s arguments about the a priori conditions on causal judgments, particularly as developed in the Second Analogy, might seem to stand or fall independently of the status of mathematical knowledge. On this line of thought, while mathematical cognition is necessary in one sense, causal—or more broadly natural—necessity arises in a wholly different manner, guided not by construction but by the application of categories to intuitively given content. Providing a Humean account of the synthetic a priori nature of mathematical judgment might then raise a challenge to Kant’s own views on mathematics, but this would not weigh on the question of whether Hume’s skeptical naturalism poses a threat to Kant’s treatment of causal judgments. That debate would have to be settled on the basis of the arguments in

the Analytic of the first *Critique*, and would essentially return us to the impasse noted at the beginning of the paper.

However, given the enormous controversy and endless debate that surrounds the question of whether Kant actually succeeds in responding to Hume's skeptical naturalism, it is also not clear that the Kantians can claim any form of victory on the causal front, especially if a key bit of favorable evidence—the synthetic a priori nature of mathematics—is stricken from their case. Indeed, perhaps Kant himself recognized something like this worry, since, as we have seen, his main complaint against Hume is *not* that his skepticism about causation is ungrounded, but that an unpalatable *consequence* of Hume's general view would be to make mathematics analytic. Hume's grave error, as Kant points out in the *Prolegomena*, lay in his failure to see the synthetic a priori nature of mathematics, and had he been forced to accept this,

he would by no means have been able to found his metaphysical propositions on mere experience, for otherwise he would have had to subject the axioms of mathematics to experience as well, which he was much too reasonable to do. The good company in which metaphysics would then have come to be situated would have secured it against the danger of scornful mistreatment; for the blows that were intended for the latter would have had to strike the former as well, which was not his intention, and could not have been. (4:273)

As Kant presents it, any attack on metaphysics broadly—and causation in particular—that Hume might mount would also have the unintended consequence of assailing mathematics as well, since the blows that strike one would also find a target in the other. This strongly suggests that far from taking causal and mathematical necessity to occupy different realms,²⁹ Kant in fact sees them as intimately linked, and maintains that it was Hume's great error not to recognize this connection. But for Hume, the lesson to be drawn from mathematics is precisely the reverse of what Kant proposes: the synthetic a priori nature of mathematical cognition shows that it cannot also be found in the realm of metaphysics, and experience alone can provide the “customary” ground for our causal claims.

Even more tellingly, Kant's posing of the main question of the first *Critique*—how are synthetic judgments possible a priori?—is, as we have seen, framed against the alternative of Hume's skeptical naturalism, and specifically his view that causal judgments as “an a priori proposition is entirely impossible, and according to [Hume's] inferences everything that we call metaphysics would come down to a mere delusion of an alleged insight of reason into that which has in fact been borrowed from experience and from habit has taken on the appearance of necessity” (B20). Hume is led to this skepticism about causal necessity, Kant continues, only

because he failed to see that “according to his argument there could also be no pure mathematics, since this certainly contains synthetic a priori propositions, an assertion from which his sound understanding would surely have protected him” (B20). Kant’s charge against Hume crucially depends on a *univocal* notion of synthetic a priori propositions in both the natural and the mathematical realms, since by Kant’s lights Hume failed to realize that his denial of necessity in causal judgments would also have the malign effect of denying the same necessity holding in the case of mathematics. At the very least, then, Kant himself seems to *assume* that the same standards of synthetic a priori cognition apply univocally to causal and mathematical judgments.³⁰

So, speaking strictly, while one might argue that the nature of mathematics does not directly bear on the arguments of the Second Analogy, Kant’s claim that only the Critical philosophy can adequately account for the synthetic a priori status of mathematical knowledge certainly provides a context in which the arguments about causality are developed. Kant’s strategy as a whole, at least as described in the B-Introduction and the *Prolegomena*, depends on accepting that both mathematics and natural science involve synthetic a priori judgments, and while an independent argument might be made for the necessity of scientific principles, it is, to put it bluntly, much easier to make a case for the synthetic a priori status of mathematics.³¹ In both the first *Critique* and the *Prolegomena*, Kant argues that pure mathematics and pure natural science are “actually given” (B20), but Hume would likely see no reason to accept that the latter actually contains the kind of “objective necessity” that Kant attributes to it; indeed, one might here take Kant to be begging the question by simply assuming that science already contains synthetic a priori cognition. It is only when one takes mathematics and science to be grounded in the *same* cognitive foundation that Kant’s project of explaining how synthetic a priori judgments are possible can be undertaken. To remove this element of Kant’s argument—or, perhaps, to emphasize the crucial difference between mathematics and causal judgments—would pose a serious threat to the overarching success of the Critical project, since it might preclude entrance into the realm of synthetic a priori judgments.

The question, then, turns on how closely mathematics stands to causal judgments, and whether the grounds and methods of one can be distinguished from the other. Here, Kant might respond that while there are of course important differences between the two kinds of cognition, they do share some key similarities, such that the nature of the former can provide a guide for the possibility of the latter. Both employ a priori concepts, and each also involves the pure forms of intuition, although these are deployed differently in mathematics and in the causal principle. For Kant, then, both traffic in synthetic a priori judgments, and this is enough to secure the claim that the status of mathematics provides evidence for the possibility of the synthetic a priori nature of the causal principle.

Are the similarities in the ways that both mathematics and the causal principle use a priori resources more important than the differences in the methods that each employs? There is no clear way to settle this question, especially since it is not something that Hume ever addresses, but it does bring to light a difficult challenge that Kant must face: how many differences between mathematics and causal judgments can be countenanced before the connection between them is attenuated or even severed? Kant seemingly has to insist that despite all of their methodological differences, mathematics and the causal principle are at root grounded on the a priori foundation of pure forms of cognition, but if this is to be Kant's position, then he stands in grave danger of begging the question against Hume about both mathematics and causality, since the similarities Kant needs between the two fields require granting the roles of a priori concepts and pure forms of intuition in each. Clearly these are not positions that Hume would recognize or accept, so the burden lies on Kant to show that they are necessary elements for any account of mathematics that recognizes its *synthetic* a priori nature. But—assuming that the conjectural story developed here is plausible—Hume can claim to have provided an account of mathematics that meets Kant's standards of being a priori and synthetic *without* appeal to forms of intuition or a priori concepts. Far from failing to see the real nature of mathematical judgment, it seems that Hume has the resources to develop a rival view of mathematical necessity that can stand as an alternative to Kant's own position, and does not require the cognitive apparatus that Kant thinks reveals the possibility of synthetic a priori causal judgments more generally. And, if this is the case, then Hume's skepticism about causal judgments stands in no danger of lopping off the province of pure mathematics, or reducing it to mere contingency.

Although I will not pursue the point here in any depth, I think this raises an interesting challenge to Kant's broader transcendental project. If we take Kant's general argumentative strategy to be one of showing that *only* his critical idealism is capable of explaining certain fundamental phenomena—he is, after all, making claims about the “necessary conditions on the possibility of cognition”—then it seems that a plausible alternative to any of his positions would threaten the transcendental nature of Kant's argument. Kant might provide us with a sufficient account of mathematics, but if the Humean view developed here is sustainable, then it is not at all clear that Kant can continue to claim that he alone has set out *necessary* conditions on the possibility of mathematics, and, by extension, causal judgments. Hume's alternative, if it can preserve the synthetic a priori character that Kant attributes to mathematics, might then offer a much broader threat than simply raising a worry about the status of arithmetic and geometry.

This in turn might suggest that the real force of Hume's alternative account of synthetic a priori judgments is best aimed not directly against Kant's account in the *Analytics* about how categories apply to intuitive content, but rather against

the arguments in the Aesthetic about space and time as pure forms of intuition. It is a contentious question whether Kant's claims in the *Critique* about space and time rest on an appeal to geometry and mathematics,³² but it is clear that he sees the possibility of mathematics resting on pure intuitions, and that this allows him to reject alternative accounts of space and time on the grounds that they cannot explain the synthetic a priori nature of mathematics: the relational view of space and time, for example, "can neither offer any ground for the possibility of a priori mathematical cognitions (since they lack a true and objectively valid a priori intuition), nor can they bring the propositions of experience into necessary accord with those assertions" (A40/B57).³³ If Hume's account of mathematics preserves its synthetic a priori nature *without* being based on pure intuitions, then this might be used to cast doubt on Kant's argument that only his transcendental idealism is capable of explaining the synthetic a priori grounds of mathematics, which in turn might undermine the arguments given in favor of treating space and time as pure forms of intuition.³⁴ And, given the central role that these forms play in section 26 of the Transcendental Deduction,³⁵ this might provide an indirect way of calling into question Kant's project in the Analytic of the first *Critique*. Again, this is not wholly dispositive, but it raises a further worry about Kant's dismissal of Hume on the grounds that he failed to see an essential feature of mathematics.

So, while Kant proposes that Hume must confront a predicament about the nature of mathematical necessity, Hume in turn can challenge Kant with a different dilemma: either concede that the constructive methods of mathematics do not tell us anything about the necessity of causal judgments, or beg the question by simply assuming that mathematics requires pure forms of intuition. In either case, Kant's frequent claims that had Hume only recognized the synthetic a priori nature of mathematics he would have grasped the foundations of the critical philosophy ring hollow. And, his strategy of motivating the transcendental project by appeal to the necessity of mathematics likewise seems to be thwarted by Hume's alternative views. Of course, this is not to say that Kant lacks the means to respond to Hume's worries about the rational foundations of our causal judgments, or that considerations of necessity independent from the concerns raised about mathematics tilt the scales in Kant's favor. But it does suggest that we should be wary of the close connection Kant explicitly draws between mathematics and philosophy. On this conjectural Humean response to Kant, *turnabout is fair play*: the initial accusation that Hume neglects the synthetic a priori character of mathematics ends up returning to confront Kant, by demanding an account of how transcendental philosophy can so differ from mathematics, yet still secure necessity in the way Kant proposes. In the old debate between Hume and Kant, perhaps this is a new way of turning the tables.

NOTES

1 References to the *Treatise* are to Hume, *A Treatise of Human Nature*, ed. Norton and Norton, hereafter cited in the text as “T” followed by Book, part, section, and paragraph number, and to Hume, *A Treatise of Human Nature*, ed. Selby-Bigge, rev. by Nidditch, cited in the text as “SBN” followed by the page number. References to the *Enquiry* are to Hume, *An Enquiry concerning Human Understanding*, ed. Beauchamp, hereafter cited in the text as “EHU” followed by section and paragraph number, and to Hume, *An Enquiry Concerning Human Understanding*, ed. Selby-Bigge, rev. by Nidditch, hereafter cited in the text as “SBN” followed by page numbers.

2 For an interesting discussion along these lines, see Falkenstein, “Hume’s Answer to Kant.” Falkenstein argues that even Kant’s attempt to establish that every event has a cause does not pose any real problem for Hume.

3 Kant, *Prolegomena*, 55 [4:257]. Further references to the *Prolegomena* will be cited in the text, following the Akademie pagination.

4 Kant, *Critique of Pure Reason*, 146 [B20]. Further references to the first *Critique* will appear in the body of the paper, using the standard A/B pagination drawn from the Akademie edition of Kant’s work.

5 For similar claims Kant makes about Hume, see also, among others, B127–29; 4:313; 5:13; 20:266; 29:782.

6 Kant, *Critique of Practical Reason*, 181–82 [5:52].

7 Though in *The Life of David Hume*, E. C. Mossner notes that during his time as a student in Edinburgh, Hume received training in mathematics from the famous Newtonian mathematician James Gregory. And, as Michael Barfoot documents in “Hume and the Culture of Science,” Hume was a member of the Physiological Library at the University of Edinburgh, which provided him access to a number of mathematical texts.

8 Hume has, to put it charitably, peculiar views about geometry, stemming from his suspicions about infinite divisibility. Not much of what I will say turns on any particular facet of geometry, and I will largely leave it aside and instead focus on arithmetic and algebra as the paradigmatic fields of mathematics.

9 Rosenberg is indicative of this view when he claims that for Hume, “mathematical statements are true in virtue of the relations between the meanings of the terms. In the terminology Kant introduced, they are analytic truths—statements true in virtue of the meanings of their terms” (“Hume and the Philosophy of Science,” 81). See also Dicker, *Hume’s Epistemology*.

10 Although a full account of what Kant means by analytic and synthetic judgments would take us too far afield, the general idea is that for Kant, all concepts contain definitional marks, and analytic judgments only expose what is already present in a concept; in Kant’s terms, analytic judgments are merely explicative. In the proposition that “[a]ll bodies are extended,” for example, the concept “extension” is contained as a definitional mark in the concept “body,” and the judgment only reveals what already is implicitly present. By contrast, synthetic judgments are ampliative, because they predicate of concepts things that are not contained in them; “All bodies are green,” for instance, makes a claim about bodies that involves something more than just what

the definitional marks of “body” include. For an account of the nature of analytic judgments, and the notion of containment at work in them, see, for example, Beck, “Kant’s Theory of Definition,” and Allison, *Kant’s Transcendental Idealism*, 89–95.

11 For this reason, Henry Allison, among others, sounds a cautionary note about trying to impose the Kantian distinction between analytic and synthetic judgments onto Hume—see *Custom and Reason*, 76ff. This may be right, but it also threatens to vitiate the force of Kant’s claims that Hume’s position fails to properly account for the synthetic nature of mathematics.

12 This is a point emphasized by Steiner, “Kant’s Misrepresentations.” For further discussion of what a Humean account of the a priori might involve, see Millican, “Sceptical Doubts,” 121–23, and Smit, “Priority, Reason and Induction.”

13 Robert Hanna’s claim that “a judgment is synthetically true if and only if it is true and its denial does not entail a contradiction” (“Kant’s Theory of Judgment”) is representative of such a view. Despite what Hanna proposes—and although Kant is certainly committed to the claim that if a judgment is analytic, then its opposite cannot be thought without contradiction—it is far from clear that he also holds the reverse, that if a judgment’s opposite cannot be thought without a contradiction, then the judgment itself must be analytic. Rather, he proposes that the principle of contradiction is the *only* ground of analytic judgments. But he never explicitly claims, as far I know, that the principle of contradiction does not apply to synthetic a priori judgments as well, or that we can conceive the opposite of a necessary judgment, whether it be analytic or synthetic, without contradiction. (On this point, in section 36 of the *Jäsche Logic*, Kant defines analytic judgments as “propositions whose certainty rests on *identity* of concepts (of the predicate with the notion of the subject),” and synthetic judgments as “propositions whose truth is not grounded on identity of concepts”—contradiction is not part of either definition, nor of the notes that follow (*Lectures on Logic*, Akademie pagination 9:111, 606–607 in the Cambridge edition). Given, say, that $5 + 7 = 12$ is a necessary truth, its opposite could be viewed as contradictory, which is supported by Kant’s claim in the B-Introduction that “the inferences of the mathematicians all proceed in accordance with the principle of contradiction (which is required by the nature of *any* apodictic certainty)” (B14; emphasis added). Likewise, Kant notes that “the proposition that no predicate pertains to a thing that contradicts it is called the principle of contradiction, and is a general though merely negative criterion of *all* truth” (A151/B190; emphasis added), and he continues that in the synthetic part of cognition, “we will, to be sure, always be careful not to act contrary to this inviolable principle [of contradiction]” (A152/B191). Hanna claims that here Kant is providing “universal, necessary and sufficient semantic condition for the truth of analytic judgments, namely that a judgment is true if and only if its denial entails a contradiction,” but it is important to note that at A150/B191, Kant only speaks of universal and sufficient conditions, but not necessary ones. This allows, I take it, that while synthetic a priori judgments must abide by the principle of contradiction, this does not provide an *explanation* of their necessity, which must be traced to intuition; analytic judgments, by contrast, are grounded solely in the principle. But, the necessity of *both* analytic and synthetic a priori judgments suggests that the opposite of neither can be conceived without a contradiction. This is by no means intended as the final word on the issue, but fortunately, given the different ways that Hume and Kant treat contradiction, I do not think that the case made here on Hume’s behalf stands or falls with the question of

whether only the opposite of synthetic judgments are contradictory, in Kant's logical sense of the term. For further discussion of this point, see also Coleman, "Is Mathematics for Hume Synthetic A Priori?"

14 To put the point slightly differently, Hume takes the causal principle, as expressed in T 1.3.3 (SBN 78–82 and EHU 4.1ff; (SBN 25ff), to be a matter of fact, and Kant's account of the causality in the Second Analogy purports to show that the causal maxim is not itself empirically derived. The issue, in other words, is about whether causality has an a priori element, rather than whether it can be seen as analytic.

15 To draw an example from David Owen, the proposition " $3467 = 2895 + 572$ " is not intuitive (in the sense that its truth can be immediately seen), but rather requires a demonstration, in which "the idea of 3467 is seen to stand in the relation of equality to the idea of $2895 + 572$, because we can match each unit in 3467 with a unit in $2895 + 572$. This matching is a matter of providing the relevant intermediate ideas," which would involve a chain of relations starting with $3467 = 3466 + 1$ and ending with $2896 + 571 = 2895 + 572$ (*Hume's Reason*, 95–96). Although Owen does not put it in these terms, these demonstrative steps seem similar to Kant's notion of calculation, and take us beyond mere definitions.

16 For a treatment of this issue, see among others Atkinson, "Hume on Mathematics."

17 What this construction involves is a rather obscure question, though fortunately I do not think it needs to be addressed in any detail here. For good accounts of Kant's position, see, among many others, Friedman, *Kant and the Exact Sciences*, and Shabel, *Mathematics in Kant's Critical Philosophy*.

18 Given Hume's Separability Principle (see, for example, T 1.1.3.4; SBN 8), presumably we cannot form a wholly distinct idea of 5, apart from noting the "numerical" features of the five fingers, and so on. Rather, in grasping numbers we would simply attend to the quantitative properties that sets of things share with other things.

19 As an aside, the difficulty with forming clear ideas of large numbers might pose a problem for Hume's account of abstraction, especially if we follow Don Garrett's suggestion that Hume grounds it in "revival sets" of particular ideas (see *Cognition and Commitment*, chap. 3). It is hard to see how my idea "12,345" could pick out a particular idea composed of 12,345 elements (and how this set could be perceptually distinguished from one with 12,346 elements), or what the revival set of such an idea would be. For this reason, I have cast Hume's argument in terms of distinctions of reason, rather than his general account of abstract ideas.

20 As Coleman puts the point, "mathematical concepts, like all other abstract ideas, are for Hume a posteriori. Nevertheless, Hume also maintains, without contradicting the forgoing claim, that necessary, and hence a priori, relations may hold between these concepts. If the distinction is made between concepts and the relation between concepts, Hume need not be obliged to maintain that mathematics is an a posteriori science, since the principles of mathematics rest solely on a priori relations between the empirical mathematical concepts" ("Mathematics," 123).

21 See Owen, *Hume's Reason*, 95–96.

22 The judgment itself—which stands as a complex idea—and not just the simple ideas that make it up, would be pure.

23 The “purity” of the mathematical ideas here need not rest on the claim that they are wholly distinct from the sets of things that represent them; rather, the distinctions of reason on which they rest allow us to attend to the numerical elements within impressions. This, of course, might seem to mark a point of divergence from Kant, but as we shall see very shortly, Kant’s account of construction—at least as developed by Shabel—in fact draws Kant closer to Hume’s dependence on something similar to a distinction of reason.

24 A problem perhaps remains, for Hume’s claim that “the necessity, which makes two times two equal to four, or three angles of a triangle equal to two right ones, lies only in the act of the understanding, by which we consider and compare these ideas; in like manner the necessity or power, which unites causes and effects, lies in the determination of the mind to pass from the one to other” (T 1.3.14.23; SBN 166) might give us pause here, since the necessity that attends both mathematical and causal judgments would arise only from an act of the understanding. Perhaps there is a sufficient enough difference between how we “consider and compare” ideas, on the one hand, and “pass from the one to the other,” on the other hand, to allow for an argument that the former approaches something like Kant’s notion of objective necessity, but this would at best be a difficult case to make, and I concede that it is not easy to read Hume as directly endorsing an account of objective necessity similar to the one Kant proposes. Still, to modify a line of thought from the “New Hume,” we might see Hume as essentially claiming that the account of necessity in terms of felt determinations of the mind does not preclude the *possibility* of an objective necessity that attends mathematical judgments. We do not have a direct idea of this necessity, but we can still specify what it would be like to be acquainted with it—there are standards, as it were, that objective necessity would have to meet, but our cognition does not rise to their level. (I am grateful to an anonymous referee for pointing out this problem, and for suggesting the general structure of the response on offer here. For a related discussion of the role that rational standards might play in Hume’s system, see Thielke, “Apostate Rationalism.”)

25 Shabel’s point here concerns geometric figures, which might seem to stand in tension with Hume’s qualms about geometry as an a priori discipline, but Shabel convincingly argues that the same manner of construction also applies to Kant’s account of algebra. See Shabel, *Mathematics in Kant’s Critical Philosophy*, section 3.3.

26 In addition to Coleman, Atkinson, and Steiner, see also De Pierris, “Hume on Space,” and Echelbarger, “Hume on the Objects of Mathematics.” The account on offer here emphasizes what I have called the constructive aspect of Hume’s view, in a way that the other papers do not.

27 Hume need not be given the final word here, since Kant might be able to provide independent reasons for thinking that causal judgments are also grounded in synthetic a priori principles. The point is only that the initial charges against Hume’s supposed failure to see the synthetic a priori nature of mathematics do not have the effects Kant intended.

28 In the *Treatise*, Hume does allow that the necessity that makes two times two equal to four “lies only in the act of the understanding, by which we consider and compare these ideas; in like manner the necessity or power, which unites causes and effects, lies in the determination of the mind to pass from one to the other” (T 1.3.14.23; SBN 166), which might imply that for Hume there *is* a univocal notion of necessity at work in both

mathematics and causality. But while the ground of necessity is located in the operations of the mind, the *structure* of the relations of ideas in mathematics seems crucially different from than those at work in causal judgments. The exactitude found in the demonstrations is not present in causal inferences, and this marks the difference that Hume could use against Kant. And, even if Hume does advance a univocal account of necessity, this alone would not serve Kant's purposes, since he likely would not accept the kind of "habitual" notion that Hume proposes.

29 In the "Discipline" section of the first *Critique*, Kant claims that the methods of mathematics and philosophy must be kept distinct, but his point there is that the results that each endeavor produces are all located in the "field of pure a priori cognition," of which the "most considerable province" is held by pure mathematics (4:272).

30 The same holds of Kant's claims in the *Prolegomena* about Hume's error arising from his failure to recognize that his attacks on metaphysics would also mean undermining mathematics, the "most considerable province" of pure a priori cognition" (4:272).

31 Admittedly, this is at best indirect evidence, but it is interesting that Hume's name is never invoked in the Second Analogy, which would be especially surprising if Kant's main response to Hume—one that ostensibly would be independent of the dispute about mathematics—is to be found there. And, even the Introduction to the *Prolegomena*, where Hume's skepticism is famously described as rousing Kant from his dogmatic slumbers, never promises to provide a direct refutation of Hume's specific attacks on the rational foundations of causality; rather, it casts the issue in terms of responding to the "Humean problem in its greatest possible amplification" (4:261), which presumably would *include* Hume's treatment of mathematics.

32 See, for instance, Allison, *Kant's Transcendental Idealism*, 116–18, for an argument that Kant's view does not depend on the truth of Euclidean geometry.

33 The same point is made in the *Prolegomena*, where Kant notes that mathematics "must be grounded in some pure intuition or other" (4:281), and that "pure intuitions a priori indeed actually do underlie mathematics, and make possible its synthetic and apodictically valid propositions; and consequently our transcendental deduction of the concepts of space and time at the same time explains the possibility of pure mathematics" (4:285).

34 The question of what exactly a pure intuition involves is important here, since in some sense the claims developed above about Hume's account of numbers as derived from distinctions of reasons might also require positing a kind of "pure" content that seemingly approaches Kant's notion of a pure intuition. There is, admittedly, a fine line to tread here, since Hume's treatment of mathematics must account for the same kind of purity that Kant posits, without succumbing to Kant's claims that *only* his view is capable of explaining it. Although a full discussion of this issue is not possible here, provisionally I think that the Humean must insist that the unity of "pure" intuitive content—which is taken to be grounded on distinctions of reason—does not derive from the formal features of sensibility, and is not structured by the application of the categories. If this case can be made out, then the connection Kant proposes between the forms of intuition and the categories would not meet the transcendental standards he requires. (Here I am grateful to the same anonymous referee for raising this point, and providing the outlines of a possible response.)

35 There, Kant notes that “we have forms of outer as well as inner sensible intuition a priori in the representations of space and time, and the synthesis of the apprehension of the manifold of appearances must always be in agreement with the latter, since it can only occur in accordance with this form” (B160). The details are devilish here, but the general point is that a priori categories can determine intuitive content because the latter is structured by the pure forms of intuition, which provide the link between the a priori and the empirical. If space and time are *not* pure forms of intuition, then the link could not be forged, and the argument of the Deduction would fail.

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